Going for the Green: A Simulation Study of Qualifying Success Probabilities in Professional Golf[1]

Robert A. Connolly and Richard J. Rendleman, Jr.

September 20, 2010

[1]Robert A. Connolly is Associate Professor, Kenan-Flagler Business School, University of North Carolina, Chapel Hill. Richard J. Rendleman, Jr. is Visiting Professor, Tuck School of Business at Dartmouth and Professor Emeritus, Kenan-Flagler Business School, University of North Carolina, Chapel Hill. The authors wish to acknowledge with gratitude Max Grender-Jones, who provided invaluable assistance in the collection of data, Kelli Anderson of the PGA TOUR, who helped to clarify many of the rules regarding Q-School competition and PGA TOUR eligibility requirements, Ben Rosen who provided helpful comments, and Kin Lo, Steven Nahmias and other participants in the "Hole in One: Using Operations Research to Analyze Golf II" session of the 2009 INFORMS conference. Please address comments to Robert Connolly (email: Robert_Connolly@unc.edu; phone: (919) 962-0053) or to Richard J. Rendleman, Jr. (e-mail: Richard_Rendleman@unc.edu; phone: (919) 962-3188).

Going for the Green: A Simulation Study of Qualifying Success Probabilities in Professional Golf

Abstract

Each year, over 1,300 golfers attempt to qualify for the PGA TOUR through Q-School. Using simulation, we estimate the probabilities that Q-School correctly identifies high-skill golfers. We show that players with skill equivalent to the very best on the PGA TOUR would have high probabilities of qualifying, but others, equal in skill to many active PGA TOUR members, would have low odds of qualifying. We explore the impact of variations in Q-school structure on qualifying probabilities for players with different skill levels, but most of the variations that improve tournament efficiency are largely impractical.

1. Introduction

Each fall the PGA TOUR conducts the PGA TOUR Qualifying Tournament – better known as Q-School – to identify a group of players to participate on the TOUR the following season who would otherwise be ineligible. Q-School consists of four stages, Pre-qualifying, which started in 2006, and Stages 1-3. Except for the final stage, each stage of Q-School is broken into a number of competitions conducted at different tournament sites, with a portion of the field at each site continuing on to the next stage. Those who do not advance from early-stage competition are eliminated and have no opportunity to re-enter the competition until the following year. Although many professionally-untested golfers must begin Q-School in Pre-qualifying, a number of golfers enter Q-School in its later stages based on their success in prior Q-School competitions and/or positions on the PGA TOUR and affiliated Nationwide Tour money lists.

The problem that the PGA TOUR is attempting to solve through its annual Q-School competition is not unlike the problems of talent selection and employee performance evaluation routinely faced by business organizations and academic institutions, among others. But unlike the promotional and talent selection practices of most organizations, the rules of Q-School are unambiguous. Both those who are being evaluated and those doing the evaluation know the "rules of the game." Once a player is evaluated, the TOUR cannot change the outcome based on subjective or political criteria. We are unaware of any set of rules for identifying the relative rankings of high skill individuals for entry into or promotion within an organization that is cleaner and more transparent than Q-School. The question we study here is whether Q-School itself is effective in identifying the most highly skilled among its entrants.

Although it is unlikely that "the next Tiger Woods" will enter Q-School on an annual basis, one would expect a regular flow of new TOUR-capable players to enter the professional golfing talent pool; otherwise the TOUR could not sustain itself over the long run.[1] As part of the "regular flow," we estimate that players with scoring characteristics comparable to well-known professional golfers would have substantially less than a 50% chance of qualifying for the PGA TOUR through Q-School

[1] We compiled two lists of players, one for 1999 and the other for 2009, who played in at least 15 PGA TOUR events as rough proxies for TOUR membership in the two years. Both lists consisted of 203 players. Only 64 players appeared on both lists, implying that 139 players who played in at least 15 events in 1999 were not among those playing 15 or more events in 2009. This suggests a turnover of approximately 14 TOUR-capable players per year between 1999 and 2009.

if they had to start at the beginning in Pre-qualifying. Moreover, our results show that a player starting in pre-qualifying with the scoring characteristics of Phil Mickelson, generally regarded as the second-best player of the modern era, would have only an 88% chance of qualifying. At the same time, players of much lower skill can make it through Q-School and qualify for the TOUR by simply having a run of favorable random variation in their scoring.

The effectiveness of Q-School in selecting the most highly skilled players to participate on the PGA TOUR depends in large part on its ability to resolve the inherent random variation in scoring to reveal, or efficiently estimate, true player skill. Ideally, Q-School would select the most highly skilled among its participants, and luck would play a minimum role in determining those who qualify for the TOUR.

In Connolly and Rendleman (2008 and 2009), we show that golfers who win PGA TOUR events generally experience substantial favorable random variation in scoring ('good luck') relative to their levels of skill. We estimate that it takes approximately 10 strokes of cumulative abnormally favorable performance over four rounds of play to win a typical PGA TOUR event. Moreover, almost all who finish among the lowest scoring 25 to 30 experience some degree of favorable abnormal performance. If 'luck' plays a role in regular PGA TOUR competition, then clearly it should also play a role in determining those among Q-School participants who are successful and unsuccessful qualifying for the TOUR.

Although no practical qualifying mechanism that the PGA TOUR could devise could eliminate the luck factor entirely, it is instructive to know just how well the current structure works. In this study we employ simulation to help answer this question. Our simulations suggest that among the most highly skilled players in a given year's Q-School competition, many will not succeed in qualifying for the TOUR due to unfavorable random variation in scoring (Type I errors), and many players of substantially lower skill will qualify (Type II errors) due to favorable random variation. Of course, the tradeoff between Type I and II errors is what drives the design of all organizational promotion activities. Unlike Q-School, however, quantifying these potential errors in other organizations is particularly difficult, especially when the rules of promotion are less clear and subject to manipulation in many forms. Moreover, it may be easy to identify Type II error rates in a business organization, that is, identify those who were promoted who turned out to be low skill, but identifying Type I error rates associated with high skill individuals who were incorrectly identified as low skill and forced to leave an organization can be much more problematic. In our study, however, we are able to estimate both types of error rates associated with the process used by the PGA TOUR to select high-skill golfers to join the PGA TOUR.

From a statistical standpoint, determining the relative skill rankings of professional golfers could be thought of as a problem in rank statistics. However, the complexity of the statistical problem suggests that an analytical evaluation of Q-School efficiency using rank order statistics is likely to be intractable. Q-School is conducted over four stages with new participants entering the competition in later stages based on prior performance on the PGA TOUR, on the affiliated Nationwide Tour and in previous Q-School competitions. Also, all but the final stage is conducted over multiple tournament sites, with those advancing to the next stage being determined based on players' relative rankings at the sites where they participate. Taken together, this structure suggests a simulation-based approach may be best suited to evaluate the selection efficiency of Q-School.

Although we conduct simulations of some obvious alternative tournament structures, such as assigning all players in a given stage to the same course and extending the number of rounds of competition, from a practical standpoint, some of these structures are not feasible. For example, it would be physically impossible to fit 900-1,000 Stage 1 participants on the same golf course at the same time. However, these alternative structures do provide insight into the extent to which limited rounds of competition over numerous tournament sites contribute to Type I and II errors. In no sense do we attempt to find optimal alternatives to Q-School.

The remainder of our paper is organized as follows. In the next section we review the literature of tournaments relevant to our study of Q-School selection efficiency. In Section 3, we describe the Q-School qualification process. We provide evidence on the success rates of golfers who qualify for the PGA TOUR through Q-School and the Nationwide Tour in Section 4 and find that those who have qualified for the PGA TOUR via the Nationwide Tour have been more successful.

In Section 5 we summarize observed qualifying characteristics of Q-School competition and variation in scoring, which, in turn, serve as the basis for the calibration of our Q-School simulation model. We discuss how we model individual golfer skill in Section 6. In Sections 7 and 8, we explain the general structure of our Q-school simulation model our method of simulating Q-School scoring. In Section 9, we discuss the simulation results, and in Section 10, compare the potential performance of hypothetical alternative Q-School structures. A final section presents a summary of the paper and our conclusions.

2. Related Work

Two approaches to the study of tournament structure embody very different interpretations of the stochastic element in measured tournament performance. According to tournament theory, both the level of effort competitors choose to exert and the degree of risk they choose to bear reflect the performance characteristics of others in the competition, the tournament's competitive design and the structure of tournament prizes.[2] Tournament theory predicts that deviations from one's "best efforts" level of performance reflect both a random component and an endogenous component, which, in turn, reflects the design characteristics of the tournament. Much of the focus in the theoretical literature of tournaments is on designing 'tournaments,' such as corporate and academic promotion rules, that will maximize the overall level of output of an organization over the long run.

The second line of tournament study, which draws on the statistical decision theory literature (for example, Gibbons, Olkin and Sobel (1977), Narayana (1979) and David (1988)), concerns the ability of a given tournament structure to identify the most highly skilled among its competitors. In this line of work, characterized by the recent papers of Ryvkin and Ortmann (2008) and Ryvkin (2010), players' efforts and risk-taking levels are not choice variables; tournament participants are assumed to play their best. As such, deviations of performance from a competitor's "best efforts" output are due to exogenous random factors alone. As noted by Ryvkin (2010, p. 668), "this assumption is realistic for an important class of selection situations involving human subjects, such as the final stages of recruitment tournaments and other environments where stakes are high, and significant prior investment has already been made by competitors." Examples cited by Ryvkin include formation of Olympic teams, innovation races, elections, and high level sports tournaments. And, most certainly, Q-School would also fit this characterization. In many cases, those participating in Q-School have spent years investing both time and money in skill development. For those

[2] The seminal paper on tournaments is Lazear and Rosen (1981). Useful surveys include McLaughlin (1988) and Prendergast (1999). Frick (2003) provides a summary of how tournament theory, as developed by Lazear and Rosen (1981) and Rosen (1986), applies in sports settings. Hvide and Kristiansen (2003) study contests where participants choose how much risk to take rather than the amount of effort to exert.

involved, the stakes couldn't be higher – being chosen as one of 25 participants from a pool of over 1,300 competitors to participate on the following year's PGA TOUR.[3] Moreover, unlike promotion tournaments in commonly-studied theoretical models applied to the promotion process in firms, many in Q-School have no baseline income from playing professional golf to fall back on in the event they do not qualify for the TOUR. For a golfer, earning the right to play on the tour is like finding the proverbial pot of gold at the end of the rainbow.

In an important set of papers, Ryvkin and Ortmann (2009) and Ryvkin (2010) study the selection efficiency of multiple tournament formats, contests, knockout tournaments, and round-robin tournaments. They explore the interaction of tournament structures, the distribution of player skills, and the level of noise in designing efficient tournaments. In their work, the tournament selection efficiency criterion is generally focused on identifying the 'best' player, not a *group* of the most highly skilled, as in Q-School. Based on their analytical and simulation results involving contests, binary tournaments, and round-robin tournaments, several predictions emerge. Of particular relevance to our work is the following finding: when players have similar skill levels and the random component in scoring is substantial, the optimal length of the tournament is much longer than in a setting with low noise and diverse abilities. Put in statistical terms, the sample size has to be very large before the distribution centers on the 'right' outcome.[4,5]

We now turn to a description of Q-School structure. Understanding this structure is critical to understanding our simulation design. Moreover, the structure should make it clear that an evaluation of Q-School selection efficiency could not be formulated analytically.

[3] At the end of the 2009 PGA TOUR season, average and median prize money for players who competed in 15 or more tournaments, a rough estimate of PGA TOUR membership, was $1,216,074 and $820,011, respectively, with 91 players earning over $1 million. It should be noted that a large portion of a PGA TOUR player's income can come from endorsements which, in turn, reflect success in tournament competition. Sirak (2008) provides an analysis of 2007 on-course and estimated off-course income for 50 professional golfers, 42 of which were active PGA TOUR members. Among the 42, the estimated off-course income for 27 golfers exceed their tournament prize winnings, and within this group, we compute a median ratio of off-course to on-course income of 1.72.

[4] We show that the error rates observed for the Q-School qualifying scheme could be reduced substantially by increasing the number of 18-hole rounds in the competition. However, the number of rounds required to bring a reasonable level of efficiency to the qualifying scheme is too high to be implemented in practice.

[5] Other studies on selection efficiency focus on algorithms for organizing and conducting particular tournament formats (knockout vs. contest vs. round-robin, to name a few) that will yield desired outcomes. For example, McGarry and Schutz (1997) use Monte Carlo methods to study the ability of knockout and round-robin tournament formats to put the ith player in the ith rank at the end of the tournament. Other examples of research on optimal tournament structure in this vein include Appleton (1995) and Clarke, Norman, and Stride (2009).

3. Q-School Structure

Figure 1 illustrates the various stages of Q-School competition as structured in 2008, how Q-School feeds the PGA and Nationwide Tours and how these two tours, in turn, feed subsequent Q-School competitions. In 2008, 433 players participated in four 18-hole rounds of pre-qualifying competition at six different tournament sites, and 238 of the 433 advanced to Stage 1. In Stage 1, the 238 were joined for four 18-hole rounds of play across 12 tournament sites by 713 golfers whose eligibility to begin in Stage 1 was based primarily on their success in previous Q-School competitions or their placing 71^{st} or worse on the 2008 Nationwide Tour Money List. Of the 951 Stage 1 participants, 308 advanced to Stage 2. They, in turn, were joined for four rounds of play across six tournament sites by 154 players whose eligibility to begin Q-School competition in Stage 2 was based on performance on the 2008 PGA and Nationwide Tours and various miscellaneous exemptions. Of the 462 participants in Stage 2, 124 advanced to the final stage, a competition at a single tournament site consisting of six 18-hole rounds, three each on two different courses. These players were joined by those in positions 126-150 on the 2008 PGA TOUR Money List and in positions 26-40 on the Nationwide Tour Money List along with a few others players who were given miscellaneous exemptions, bringing the total number of Stage 3 competitors to 161. The lowest scoring 25 players and ties (28 total) earned 2009 PGA TOUR cards. Those in positions 29 to 70 in the final stage (the next closest to 50) earned fully-exempt playing privileges on the 2009 Nationwide Tour.[6] All remaining players earned partially-exempt status on the Nationwide Tour, with initial preference given to those with the best scores in the final stage of Q-School competition.[7] As it turns out, only six of the 433 participants in Pre-qualifying made it to Stage 3, and among the six, the two best finishers, Joseph Sykora and Martin Piller, earned fully-exempt status on the Nationwide Tour.

Figure 1 shows that in addition to the (approximately) 50 annual PGA TOUR qualifiers from Q-School and the Nationwide Tour, the top 125 players on the PGA TOUR Money List get to return to the PGA TOUR the following year. The players in positions 126-150 on the Official

[6]The term, "fully exempt player," as it is applied to the PGA TOUR or Nationwide Tour, is a player who has earned the right to play on the tour without restriction, other than being subject to tournament priority lists based on past performance.

[7]A "partially exempt player" on the PGA TOUR or Nationwide Tour is one who is allowed to fill out a tournament field after all fully-exempt players have been given the opportunity to participate.

PGA TOUR Money List advance immediately to Stage 3 of 2008 Q-School. Even if they had not qualified for the 2009 PGA TOUR through 2008 Q-School competition, they would have received partially-exempt status on the 2009 PGA TOUR. PGA TOUR players in positions 151 and higher on the 2008 Official PGA TOUR Money List advance immediately to Stage 2 of Q-School, while the first 50 players in this group also receive partially-exempt status on the following year's Nationwide Tour. It should be noted that no official PGA TOUR member can do any worse than returning to Stage 2 of Q-School. This is a key characterization of Q-School competition in structuring our simulations.

Figure 1 also shows that players in positions 26-60 on the 2008 Official Money List of the Nationwide Tour were able to return to the Nationwide Tour in 2009 as fully-exempt members. The first 15 players in this group advanced immediately to Stage 3 of 2008 Q-School. Nationwide players in positions 61-100 earned the right to return to the Nationwide Tour in 2009 as partially exempt members. We leave the remaining observations about the relationships among Q-School, PGA TOUR and Nationwide Tour qualification to the reader. We note, however, that there are other ways of making the PGA and Nationwide Tours, which are not included in Figure 1, such as winning a PGA TOUR event during the past two years or being a non-PGA TOUR member who earns as much official prize money as the 125^{th} player on the Official PGA TOUR Money List. Figure 1 is intended to illustrate the interconnections among Q-School, PGA TOUR qualification and Nationwide Tour qualification only.

4. Do Q-School and the Nationwide Tour Identify the Best Players?

Table 1 summarizes the distribution of Official World Golf Rankings (OWGR) as of July 26, 2009 among those who qualified for the PGA TOUR through Q-School and the Nationwide Tour between 1998 and 2008. Of particular note is the high proportion, 36%, of 1998-2008 Q-School graduates (those who qualify for the PGA TOUR through Q-School competition) who fell outside the top 1,000 of the World Golf Rankings as of July 26, 2009. By comparison, only 23% of the Nationwide Tour graduates fell the top 1,000. Moreover, the higher proportion of players graduating from the Nationwide Tour who were among the OWGR's top 500 (54% vs. 41%), along with the lower proportion ranked worse than 1,000, suggests that since 1998, the Nationwide Tour has done a better job identifying the most highly-skilled players as of July 2009, despite the fact that Nationwide Tour participants did not perform as well in Q-School to begin with.

These findings suggest a non-negligible 'luck' factor associated with making it onto the PGA TOUR via Q-School, implying a relatively high Type II error rate among players identified through Q-School as highly skilled. By contrast, Nationwide Tour competition extends presently over 29 tournaments, and, therefore, it is less likely that relatively weak players will qualify for the PGA TOUR through Nationwide competition. Moreover, since favorable random variation in scoring tends to play a substantial role in qualifying for the PGA TOUR through Q-School, many highly-skilled players in Stage 3 who do not finish among those who qualify for the PGA TOUR but go on to the Nationwide Tour could be as skilled, if not more skilled, than many of the Stage 3 players who actually qualify for the TOUR. With 29 additional tournaments to be played the following year on the Nationwide Tour, many who qualify for the PGA TOUR through Nationwide competition are likely to end up being more successful in the long run than some who qualified for the PGA TOUR from Q-School ahead of them.

5. Properties of Q-School Competition and Scoring

In this section we describe general properties of Q-School qualifying success rates and scoring which, in turn, serve as the basis for calibrating our simulations of Q-School competition. Although we draw some general inferences from the data, none are supported by tests of statistical significance, since the inferences drawn are not the primary focus of this study.

5.1. Qualifying Success Rates

Table 2 shows how the players who participated in Q-School from 2006 to 2008 advanced through its various stages. All values within the table, as well as Tables 3 - 6, reflect the combined 2006-2008 data. Over this period, 1,191 players entered Q-School at the Pre-qualifying stage.[8] Only 13 of the 1,191 reached Stage 3, but none qualified for the PGA TOUR, and very few who entered in Stage 1 earned their TOUR cards (25 of 2,099).

Table 3, derived from the entries in Table 2, shows qualifying success rates by stage of play,

[8]If a player entered Q-School in Pre-qualifying in each year, 2006-2008, he is counted as three players.

conditional upon Q-School entry stage. By design, 53.7% of the participants in Pre-qualifying advanced successfully to Stage 1. Stage 1 participants, in turn, consist of two groups, those who started in Pre-qualifying and those who started in Stage 1. Among the Stage 1 participants who started in Pre-qualifying, 21.0% (134 of 639) advanced to Stage 2, while 38.6% of the Stage 1 participants who started Q-School in Stage 1 advanced to the next stage. We note that one of our main objectives in structuring simulations of Q-School competition is to produce qualifying success rates in simulated data that closely approximate those shown in Table 3.

The entries in Table 3 suggest that the level of skill among those who participated in a given stage of Q-School is an increasing function of entry stage. For example, only 9.7% of the Stage 2 participants who began Q-School in Pre-qualifying advanced to Stage 3. By contrast, 24.2% of the Stage 2 participants who started in Stage 1 advanced to Stage 3, and 36.6% of those who entered Q-School in Stage 2 advanced to the final stage. There is one notable exceptions to this pattern – in Stage 3 – where those who began Q-School competition in Stage 2 were more successful earning their TOUR cards than those who entered in Stage 3 (27.1% vs. 20.0%).

5.2. Mean Scoring of Q-School Participants

Table 4 summarizes mean 18-hole scores, conditional upon Q-School entry stage, normalized to mean scores in Stage 3 for players who entered Q-School in Stage 3. Without exception, in each playing stage mean scores are the lowest among players who entered Q-School in the later stages. Consistent with the qualifying success rates summarized in Table 3, this suggests that, on average, players who are allowed to bypass early-stage competition and enter Q-School in the later stages are more highly skilled than the players against whom they are competing who entered Q-School in earlier stages.

Also, players who advance in Q-School competition tend to record lower scores on average in advanced-stage competition than all players *in their same entry group* scored in earlier competition. This suggests that players who advance in Q-School are more highly skilled *as a group* than those in their same entry group who do not advance. However, differences in average scores from one stage to the next for a given entry group could also reflect the relative difficulty of courses rather than differences in skill.

5.3. Mean Scores Adjusted for Relative Round and Course Difficulty

We have insufficient player connections within the Q-School data to estimate simultaneously both player skill and the relative difficultly of each round-course combination (interaction) as in of Connolly and Rendleman (2008). For example, in 2008, we cannot estimate the inherent difficulty of round 1 of Pre-qualifying at Kinderlou Forest Country Club (Course Rating of 76.6) relative to round 3 of Stage 1 played at Martin Downs (Course Rating of 73), while simultaneously estimating the skill of the players participating on those courses.[9] If Pre-qualifying entrants who reached Stage 1 recorded lower scores on average than all players who competed in Pre-Qualifying, is it because those who advanced to Stage 1 are better players, or is it because a portion of Stage 1 competition was conducted on a relatively easy course? We simply cannot tell.

To control for relative course difficulty and playing conditions, we compute the difference between each 18-hole score and the mean score for the same round on the course on which the round is played. We refer to such scores as "round-course-adjusted" scores. For each playing stage, we then *("playing stage-")* normalize the mean of the round-course-adjusted scores to the mean associated with players entering Q-School in that particular playing stage. Table 5 summarizes the playing stage-normalized, round-course-adjusted mean scores for years 2006-2008 combined. Again, we observe that in each playing stage, mean (round-course-adjusted) scores are the lowest among players who entered Q-School in the later stages.

5.4. Variation in Round-Course-Adjusted Scores

Table 6 summarizes standard deviations of round-course-adjusted scores for each playing stage/ entry stage category. The standard deviations of round-course-adjusted scores within each playing stage category exhibit little variation conditional upon entry stage. Generally, standard deviations for each entry group tend to be lower as players within the group advance from one stage of Q-School to the next.[10]

[9] According to the USGA (www.usga.org), a USGA Course Rating is the evaluation of the playing difficulty of a course for scratch golfers under normal course and weather conditions. It is expressed as the number of strokes taken to one decimal place (72.5), and is based on yardage and other obstacles to the extent that they affect the scoring difficulty of the scratch golfer. A male scratch golfer is a player who can play to a Course Handicap of zero on any and all rated golf courses.

[10] As noted in Connolly and Rendleman (2008, p. 82), and as we also show in Section 8.2, standard deviations of round-course-adjusted scores tend to be lower for players with lower mean scores (i.e., higher skill players play with more consistency). Therefore, if the more highly-skilled players within an entry group are advancing through

The standard deviation of round-course-adjusted scores is much higher in Pre-qualifying than in the other stages of play. We believe that this high standard deviation reflects more than just greater variation in round-to-round scoring among Pre-qualifying participants. Instead, it appears that Pre-Qualifying consists of at least two distinctly different groups of players – those who are legitimate candidates to participate in professional golf competition and those who have no realistic chance of qualifying for the TOUR.

In our simulations, we are unable to replicate the pattern of standard deviations shown in Table 6 in the Pre-qualifying stage without modeling pre-qualifiers as a bimodal group, with 75% of the pre-qualifiers being modeled as 'legitimate' players and 25% consisting of a second group whose scores, on average, are 5.38 strokes higher. We note, however, that the qualifying success rates that we estimate in our simulations are virtually the same, whether we assume that 25% of the players in Pre-qualifying are among those in the second group or whether there is no second group at all.

6. Estimating Skill of Individual Q-School Participants

The primary focus of this study is to estimate the probabilities associated with individual players qualifying for the PGA TOUR through Q-School as a function of their skill levels. In 2008, 1,339 players recorded 8,265 scores in Q-School competition, an average of 6.17 scores per player. Unfortunately, this represents an insufficient number of observations to estimate individual skill with any degree of precision. Moreover, due to the nature of Q-School competition, there is inherent bias in individual player sample mean scores relative to true mean scores. Although many who advance in the competition are likely to advance for being more skilled compared with those who do not advance, others may advance for having experienced more favorable random variation in their scoring. Similarly, those who do not advance are likely to be less skilled, but also are likely to have experienced unfavorable random scoring variation. Therefore, in a given stage, as a group, the sample mean scores of players who advance to the next stage of competition should be biased downward relative to true mean scores, and the mean scores of those who do not advance should be biased upward.

In addition, we would like to estimate the probabilities that players with skill comparable to

Q-School, standard deviations of round-course-adjusted scores for a given entry group should be lower in the later stages of Q-School competition.

successful PGA TOUR players would make it through Q-School if they had to begin Q-School competition in Pre-qualifying. There is simply no way to draw such inferences by studying the 2006-2008 Q-School data in isolation without also connecting the Q-School data to scoring data characteristic of regular players on the TOUR.

The key to making this connection is through the regular PGA TOUR players who participate in Q-School after unsuccessful PGA TOUR seasons. Defining a regular PGA TOUR player as one who participated in at least 15 events during the PGA TOUR season immediately preceding entry to Q-School, we find that 40, 33, and 47 such players entered Stage 2 of Q-School in years 2006-2008, respectively, and 21, 20, and 16, respectively, entered Stage 3.[11] Using the statistical methodology of Connolly and Rendleman (2008) to estimate the scoring characteristics of regular TOUR players over the 2004-2008 PGA TOUR seasons, including those who had to go back to Q-School, we can link the scoring characteristics of Q-School participants coming off the PGA TOUR to those of more successful TOUR players such as Tiger Woods and Phil Mickelson who do not participate in Q-School.

Using the Connolly and Rendleman (2008) model, we estimate cubic spline-based time-varying skill functions for a group of professional golfers representative of active PGA TOUR participants during the five-year period 2004-2008. Simultaneously, the model accounts for random variation in scoring due to differences in relative round difficulty (round-course effects) and the propensities of individual players to perform better or worse on certain courses (player-course effects). (To conserve space, we refer the reader directly to Connolly and Rendleman (2008) for a description of the statistical methodology, its relationship to previous literature on performance measurement in golf and other athletic contests, and its efficiency relative to alternative modeling structures.) We note that the model does not take account of specific information about playing conditions (e.g., adverse weather as in Brown (2010), pin placements, morning or afternoon starting times, etc.) or, in general, the particular conditions that could make scoring for all players more or less difficult, when estimating random round-course effects. Nevertheless, if such conditions combine to produce abnormally high or low scores in a given 18-hole round, the effects of these conditions should be

[11] We are not concerned with whether a player was an official member of the PGA TOUR during the PGA TOUR season immediately preceding his entry to Q-School but, instead, whether the statistical properties of his play were characteristic of those of regular TOUR players who entered Q-School. Therefore, some players so identified might not have been official members of the TOUR, while others who participated in less than 15 events could have actually been official PGA TOUR members.

reflected in the estimated round-course-related random effects.[12]

To fit the model, we collected individual 18-hole scores from the golf section of Yahoo! Sports for every player in every stroke play event on the PGA TOUR for years 2004-2008, a total of 96,264 scores distributed among 1,432 players. After limiting our sample to players who recorded more than 90 scores, as in Connolly and Rendleman (2008), the resulting sample consists of 83,823 18-hole golf scores for 303 active PGA TOUR players over 231 stroke-play events. Most of these omitted players were not regular PGA TOUR players. For example, 528 of the omitted players recorded only two 18-hole scores, and 906 of the 1,432 players in the sample recorded 10 or fewer scores. By excluding these players, we maximize the power of the statistical model and minimize potential distortions in estimating the statistical properties of golf scores of regular players on the TOUR.

Estimated random round-course effects in the 2004-2008 PGA Tour sample range from -3.76 to 7.82 strokes per round (compared with -3.92 to 6.95 strokes per round in the 1998-2001 sample), implying almost a 12-stroke difference between the relative difficulty of the most difficult and easiest rounds played on the Tour during the 2004-2008 period. Estimated random player-course effects range from -0.197 to 0.166 in the 2004-2008 sample and from -0.065 to 0.044 strokes per round in the 1998-2001 sample, too small to have a meaningful impact on the overall scores in a typical 72-hole PGA Tour event.

We have identified the players among the 303 who participated in Stages 2 and 3 of Q-School during the 2004-2008 period. Using the estimated skill functions for these players during the PGA TOUR seasons immediately preceding their entry to Q-School, we are able to build scoring distributions characteristic of regular PGA TOUR players who entered Q-School in Stages 2 and 3. We limit these skill functions to the portions of individual player splines estimated over the PGA TOUR seasons immediately preceding Q-School entry, since sub-standard single-season performance is the determining factor in a PGA TOUR player having to return to Q-School. Inasmuch as skill functions for these players are estimated simultaneously with those of the remaining players in the 303-player sample, we are able to connect the scoring characteristics of those among the 303 who

[12] Round-course effects, with similar justification, are also estimated in Berry, Reese and Larkey (1999) and Berry (2001). A similar approach is taken by Caulkins et. al (1993) in estimating on-time performance of airlines, where empirically observed on-time performance rates reflect "not just the difficulty of flying into [a given airport], but also the skill of the airlines that serve it." (pg. 713)

entered Q-School between 2004 and 2008 to those of Tiger Woods and other highly-skilled players on the TOUR.

From this point forward, we will refer to two separate 18-hole scoring samples. The first, the "2006-2008 Q-School sample," is the sample of scores in Q-School competition covering the 2006-2008 period whose properties are summarized in Tables 2 through 6. The second is the separate sample of scores associated with 303 players in regular PGA TOUR competition, some of whom also participated in Q-School, which we will refer to as the "2004-2008 PGA TOUR sample."

7. Q-School Simulation Structure

7.1. Creating a Prototypical 2006-2008 Q-School Structure

In our simulations, we employ a Q-School structure that reflects the number of total and successful participants by stage of play observed in actual Q-School competition over the 2006-2008 period. Each simulation consists of 1,000 trials, with each trial representing the simulation of a single Q-School competition.

Although not shown in the Table 2, during 2007 and 2008, Pre-qualifying was conducted at six different tournament sites, Stage 1 was conducted at 12 sites, Stage 2 was conducted at six sites, and only one site was used for the final stage, although competition in the final stage was conducted on two courses, with each competitor playing half of his rounds on each course. We will refer to this as a 6-12-6-1 venue structure. The venue structure in 2006 was slightly different. Pre-qualifying competition was conducted at four tournament sites, and Stage 1 was conducted at 11, but, otherwise, the structure was the same.

In our simulations, we employ a prototypical Q-School structure that reflects the average number of total and successful participants by stage of play in the 2006-2008 data, spread over a 6-12-6-1 venue structure. We then attempt to calibrate the parameters of assumed scoring distributions employed in the simulations so that the simulations produce qualifying success rates and standard deviations of round-course-adjusted scores by playing stage/entry stage that closely match those same properties as observed in the 2006-2008 empirical Q-School sample.

Table 7 summarizes the Q-School structure used in calibrating our simulations. Pre-qualifying is spread over six tournament sites, with approximately 66 players participating at each site and

35 qualifying for Stage 1 competition. In stage 1, approximately 26 of 76 participants per venue advance to Stage 2. Stage 2 is comprised of approximately 78 players per venue, with approximately 21 per venue advancing to Stage 3. The single tournament site in Stage 3 hosts 163 players, and 31 advance to the PGA TOUR. We do not spread the Stage 3 players over two different courses, as is the case in actual Q-School competition.

7.2. Player Groups

Although not shown in Table 7, an average of 40 players per year who entered Q-School in Stage 2 were regular PGA TOUR players (defined in our study as having participated in at least 15 PGA TOUR events during the season immediately preceding their entry to Q-School), and an average of 19 regular PGA TOUR players entered Stage 3. Therefore, in our simulations, we assume that 40 Stage-2 entrants and 19 Stage-3 entrants come directly from the TOUR. We assume further that all remaining players in Q-School competition are non-PGA TOUR players. This results in the following six distinctly different player groups, with an indication of the number of players per group employed in the simulations:

1. Non-PGA TOUR players entering Q-School competition in Pre-qualifying (397 players, see Table 7)
2. Non-PGA TOUR players entering Q-School competition in Stage 1 (700 players)
3. Players entering Q-School competition in Stage 2 coming directly from the PGA TOUR (40 players)
4. Non-PGA TOUR players entering Q-School competition in Stage 2 ($151 - 40 = 111$ players)
5. Players entering Q-School competition in Stage 3 coming directly from the PGA TOUR (19 players)
6. Non-PGA TOUR players entering Q-School competition in Stage 3 ($38 - 19 = 19$ players)

As described in Section 8, the statistical properties of scoring for the two groups of players coming directly from the PGA TOUR (groups 3 and 5, above) reflect the scoring characteristics of players of this type as estimated in the 2004-2008 PGA TOUR sample during the PGA TOUR seasons immediately preceding their Q-School entry without alteration. The statistical properties of scoring for non-PGA TOUR players entering Stage 3 are based on a potential modification (described in Section A of the Appendix) of the scoring properties of Stage 3 players coming directly from the TOUR. The statistical properties of all other non-PGA TOUR player groups are based on potential modifications of scoring distributions associated with Stage 2 players coming from the TOUR (also described in Section A of the Appendix).

7.3. Simulation of Q-School Competition

If a player enters Q-School in Pre-Qualifying and makes it to Stage 3, he will have participated in 18 rounds of golf, four rounds each in Pre-qualifying through Stage 2 and six rounds in Stage 3. Using the procedure described in Section 8, we generate 18 random 18-hole scores for each player in simulated Q-School competition but use only the portion of the scores so generated that are relevant to a player's stage of play. For example, if a player enters Q-School in Stage 3, we would use only the last six of his 18 randomly generated scores.

The following steps, which reflect the prototype Q-School structure summarized in Table 7, describe the structure of simulated Q-School competition:

1. Select 397 players at random for Pre-qualifying and generate 18 random 18-hole scores for each player using the procedures described in Section 8.
2. Randomly assign each of the 397 players to each of six pre-qualifying venues, with approximately 66 players assigned to each venue.
3. Compute the total of the first four randomly generated scores for each player and rank each player from lowest total score to highest total within each venue.
4. From each ranked list, select the (approximately) 36 players with the lowest total scores from each venue as venue winners who move on to Stage 1 (213 total).
5. Combine the 213 Pre-qualifying winners with 700 new Stage 1 entrants to produce a pool of 913 players for competition in Stage 1.
6. Distribute the 913 players randomly across 12 venues of approximately 76 players each.
7. Generate 18 random 18-hole scores for each new player entering Q-School in Stage 1.
8. Compute the total of scores 5-8 for each of the 913 Stage 1 players and determine venue winners who move on to Stage 2 (315 total).
9. Proceed in likewise fashion through Stage 3, using the number of new stage entrants, venues and venue winners as indicated in Table 7.
10. The lowest scoring 31 players in Stage 3 qualify for the PGA TOUR.

We explore variations on this general simulation structure in Section 9.

8. Simulated Scoring Distributions for Q-School Participants

8.1. Simulated Scoring Distributions for PGA TOUR Players Entering Q-School

When estimating cubic spline-based skill functions for each of the 303 players, we also obtain sets of player-specific residual scoring errors, denoted as θ and η. The θ errors represent potentially autocorrelated differences between a player's actual 18-hole scores, reduced by estimated random round-course and player-course effects, and his spline-predicted scores. The η errors represent θ errors adjusted for estimated first-order autocorrelation, and are assumed to be white noise.

We recognize that a player's official money winnings in a given PGA TOUR season, which, in turn, reflect his scoring, determines whether he must participate in Q-School at the season's end to qualify for PGA TOUR play the following year. Therefore, in building the simulated scoring distribution for a regular PGA TOUR player entering Q-School in Stage 2 or 3, we include only the portion (or those portions) of his fitted spline associated with the PGA TOUR season(s) in which his performance over 2004-2008 was sufficiently poor to require him to enter Q-School competition at the end of a season. Using this procedure for constructing simulated scoring distributions characteristic of players entering Q-School in Stages 2 and 3, 161 player-year combinations are represented in the Stage 2 group, and 86 are represented in the scoring distributions for Stage 3.

We do not include a player in our scoring distributions for Stages 2 or 3 if he recorded fewer than 40 scores on the PGA TOUR during the season immediately preceding his entry to Q-School. The 40-score minimum provides an indirect way of determining whether the player was a 'regular' on the TOUR. We refer to these two groups, respectively, as our Stage 2 and Stage 3 PGA TOUR samples.[13]

In each simulation trial, we assume that 40 regular PGA TOUR players enter Q-School in

[13] Even though a player's loss of PGA TOUR playing privileges is based on money winnings, not average score, the two are clearly related. In addition to playing poorly, some players could come up short on money winnings by simply not having played in a sufficient number of tournaments. Not every PGA TOUR player, or player in our sample, is eligible to participate in the majors. Some of the better "invitationals" on the PGA TOUR, such as the Arnold Palmer Invitational (formerly Bay Hill) and the Memorial Tournament, and the several World Golf Championship events on the TOUR employ eligibility requirements that restrict competition to only select groups of top players. Moreover, since tournament payoffs are highly non-linear, two players with identical average scores across the same set of tournaments could end up with substantially different money winnings. So it is possible that some regular PGA TOUR players who are required to go back to Q-School at the end of an unsuccessful PGA TOUR season are more highly skilled, and may have actually performed better in terms of scoring, than other players who are able to retain their TOUR cards. Our Stage 2 and Stage 3 PGA TOUR samples reflect this potential heterogeneity in skill among players required to "go back to school," since the two samples consist of players who actually participated in Q-School after coming off the TOUR (for whatever reason).

Stage 2 and that 19 enter in Stage 3. In simulating the scoring in Q-School of each of the 40 regular TOUR players, indexed by i, who enter Stage 2, we select a player, k, at random, with replacement, among those included in the 161-player Stage 2 PGA TOUR sample. (Throughout this section and the next, the index i always refers to a Q-School participant, and the index k always refers to a randomly selected player in the Stage 2 or Stage 3 PGA TOUR sample upon whom player i's scoring distribution is based.) We then select a point at random on the portion of player k's estimated cubic spline covering the PGA TOUR season immediately preceding his entry to Q-School as an estimate of player i's mean skill level.

To simulate random variation in scoring for Q-School participant i, we select a starting θ error at random from the entire distribution of player k's θ errors estimated over the 2004-2008 PGA TOUR sample period. We then select 28 η errors at random (with replacement) from player k's entire distribution of η errors. Using the initial θ error, the vector of 28 randomly-selected η errors, and player k's first-order autocorrelation coefficient as estimated in the Connolly-Rendleman model, we compute a sequence of 28 estimated θ errors. We apply the last 18 estimated θ errors to Q-School player i's randomly selected mean score, yielding a sequence of 18 simulated 18-hole scores for player i. We do not employ the first ten simulated θ errors in our simulated scores in order to give the autocorrelated component of residual scoring errors time to 'burn in.' Simulated scores for the 19 PGA TOUR players who enter Q-School in Stage 3 are produced in similar fashion. By construction, all of these simulated scores are 'neutral' with respect to variation in scoring due to relative round-course difficulty and a player's propensity to score better on some courses than others.

The two right-most histograms in the bottom section of Figure 2 show distributions of mean skill levels, based on 1,000 simulation trials, for PGA TOUR players entering Q-School in Stages 2 and 3, respectively. Histograms in corresponding positions in Figure 3 show distributions of actual 18-hole scores for both groups. Simulated 18-hole scores range from 60.92 to 87.53 among those shown in the Stage 2 histogram (720,000 scores total) and from 60.81 to 85.24 among those shown in the histogram for Stage 3 (342,000 scores total).

Note from Figure 3 that there is a $72.04 - 71.57 = 0.47$ stroke-per-round average scoring difference between the scoring distribution of PGA TOUR players entering Q-School in Stages 2 and 3. Therefore, in our simulations, PGA TOUR players who enter in Stage 3 have a built-in mean 0.47 stroke-per-round advantage over those who enter in Stage 2.

8.2. Simulated Scoring Distributions for Non-PGA TOUR Players Competing in Q-School

As described in Section 7.2, we build simulated distributions of scoring for all Stage 3 entrants not coming from the PGA TOUR by modifying simulated scores that would otherwise be associated with Stage 3 entrants coming directly from the TOUR. Similarly, we build simulated distributions of scoring for all Pre-qualifying and Stage 1 entrants and for those entering Q-School in Stage 2 who are not coming from the PGA TOUR by modifying simulated scores that would otherwise be associated with Stage 2 entrants coming directly from the TOUR.

The modified distributions are 'parameterized' to produce simulated qualifying success rates per playing stage/entry stage category that closely approximate those observed empirically, as summarized in Table 3, and simulated standard deviations of 'neutral' scores per playing stage/entry stage category that closely approximate the empirically-observed standard deviations of round-course-adjusted scores summarized in Table 6. We summarize the modification process and the sensitivity of our results to parameter selection in Sections A and B of the Appendix.

Panels A and B of Table 8 show how well qualifying success rates and standard deviations in the simulated data match those observed in the 2006-2008 Q-School sample. Note that there is very little difference between the simulated and empirical qualifying proportions and standard deviations.

In Panel C of Table 8, we also show playing stage-adjusted, round-course-adjusted mean scores per playing stage/entry stage category for both the simulated and empirical data. We make no attempt to match these values in selecting our simulation parameters. Although some of the differences appear to be large, they are well within the bounds of year-to-year variation underlying the entries in Table 5. Therefore, we believe that the simulations, calibrated with the parameters summarized in Table 8, reproduce the properties of the 2006-2008 Q-School sample in its most important dimensions.

The center section of Figure 2 shows simulated distributions of mean skill for all player groups derived from the Stage 2 PGA TOUR sample, while the right-most section shows scoring distributions based on the Stage 3 sample. Corresponding sections of Figure 3 show distributions of simulated 18-hole scores. One striking observation from the distributions is the higher average score of non-PGA TOUR players entering Stage 3 compared with Stage 2. However, from Table 3, we can see that once having reached Stage 3, those who entered Q-School in Stage 2 have tended to be more successful than players who began Q-School play in Stage 3. In order for our simulated Q-School results to have this same property, non-TOUR players who enter Q-School in Stage 3 must be less skilled, on average, than those who enter in Stage 2.

9. Simulation Results

9.1. Qualifying Success Rates

We split Q-School participants into groups of 25 players based on mean skill and estimate the frequency of success in Q-School for each player group across 1,000 trials of the simulation. We place the 25 most highly-skilled players per simulation trial in group 1, the next 25 most highly-skilled players in group 2, etc. through group 20. We then lump all the remaining players, those ranked 501 to 1,286, into group 21.

We break participants into groups of 25 since, as of 2008, the lowest 25 finishers and ties at the conclusion of play in Stage 3 qualify for the PGA TOUR. (In 2007 the number of qualifiers was 30 and ties.) The remainder of players in Stage 3 are eligible for Nationwide Tour membership. The next number of finishers closest to 50 qualify as "fully-exempt" members of the Nationwide Tour, eligible to play in any Nationwide Tour event the following year. The remaining Stage 3 participants become partially-exempt Nationwide players. Ideally, the 25 most highly skilled players among the 1,286 would have high probability of qualifying for the TOUR.

As shown in Table 9, which provides qualifying success rates for each of the 21 player groups, this is not the case. Only 33.0% of the 25 most highly-skilled Q-School participants finish Stage 3 among the 25 lowest scoring players. Among those finishing among the lowest-scoring 25, only eight players would have come from skill group 1.

By finishing among the lowest-scoring 75, 59.9% of the players in group 1 would have done no worse than earning fully-exempt Nationwide status, and by participating in Stage 3, 72.7% would have done no worse than earning a partial exemption on the Nationwide Tour. The remaining 27.3% would have earned nothing more than the right to enter Q-School the following year in Stage

1 (see Figure 1).

As one would expect, the qualifying success rates fall monotonically by skill group. Taking groups 1-3 together, only 35.5/75 = 47.3% of the 75 most highly-skilled players in Q-School competition would have finished among the 75 lowest-scoring players at the end of Stage 3.

Although the qualifying success rates in individual skill categories 4-21 are not high, collectively we see that many players of low skill finish among the lowest scoring 25 who would have qualified for the TOUR, many would have earned no worse than fully-exempt Nationwide status, and at least 12.1% in each of skill groups 1-20 would have done no worse than earn partially-exempt Nationwide status. In fact, 3.4% of the players ranked 501 to 1,286 (those in skill group 21) would have earned the right to play on the Nationwide Tour in some capacity, while 100% − 72.7% = 27.3% of the 25 most highly-skilled players would not have earned that right.

Note that 0.1% of the players in group 21 finish among the lowest 25 scoring players in Stage 3, equivalent to almost one player per year. Moreover, 27.0 players among the 163 Stage 3 players who, by participating in Stage 3, would earn no worse than partially-exempt Nationwide status (16.7%), would have come from those ranked 501 or worse.

9.2. Success Rates for 'The Next Tiger Woods' and Five Other Highly-Skilled Players

It is interesting to consider how well players of skill comparable to that of Tiger Woods and other high-skilled PGA TOUR professionals, as estimated over the 2003-2009 period, would fare if they had to begin Q-School in Pre-qualifying. To address this question, we replace six randomly-selected players in Pre-qualifying with scoring characteristics comparable to Tiger Woods and five other "good" players. In each simulation trial, the first "good" player is a player whose simulated score is drawn from the distribution of Tiger Wood's scores from the 2004-2008 PGA TOUR sample. The second good player is a player with scoring characteristics identical to that of a player selected randomly from skill positions 2-21 among the 303 players in the 2004-2008 PGA TOUR sample. In determining these skill positions, a player's skill is defined as the average value of his spline-based mean 18-hole score as estimated over the entire 2004-2008 PGA TOUR sample period. The third good player is selected randomly from skill positions 22-41, etc., with the sixth selected randomly from positions 82-101. While it is unrealistic to assume that a 'Tiger Woods' will come along every year, it is not unrealistic to assume that players like those in quintiles 1-5 will enter Pre-qualifying on a regular basis. Otherwise, there would be no permanent talent pool on the TOUR. Simulated mean skill levels and variation in scoring among the six good players are determined using the same sampling procedure described in Section 8.1 for PGA TOUR players entering Q-School in Stages 2 and 3, except that skill levels for these players are drawn from their entire estimated splines rather than from smaller spline segments.

Histograms showing the distributions of skill and 18-hole scores for the "good" player group are shown on the far left of Figures 2 and 3. Note that the skill distribution clearly is bimodal, reflecting that players with Tiger Woods' scoring characteristics are substantially more skilled than the other "good" players. Also note that as a group, the mean score in the "good" player group is $71.58 - 70.14 = 1.44$ strokes less than that of players in the next-most highly-skilled group, PGA TOUR players entering Q-School in Stage 3.

If Q-School is working properly, we believe that a player of Tiger Wood's ability who enters Q-School in the Pre-qualifying stage should almost always qualify to play on the PGA TOUR. Moreover, one would hope that players of skill comparable to those in skill positions 2-101 on the PGA TOUR should have little trouble qualifying. Panel A of Table 10 summarizes the probabilities of making it through the four stages of Q-School for players of Tiger's Wood's ability and for randomly selected players from quintiles 1-5. In Table 10, "making it to the next stage" from Stage 3 refers to finishing among the lowest scoring 31, the average number of qualifiers over the 2006-2008 period.

Not surprisingly, a player with skill comparable to that of Tiger Woods would earn his TOUR card 99.3% of the time. However, the qualifying success rate for those in quintile 1 is only 68.8%. Moreover, less than half of the players in quintiles 3-5 earn TOUR cards, with those in quintile 5 qualifying for the TOUR only 28.7% of the time.

It should be noted that all of the success rates shown in Table 10 are conditional upon a Tiger-type player and players in all five quintile groups playing simultaneously in the Q-School competition. Therefore, the success rates for each of the six player groups should be lower than if no other groups were included. However, the estimated success rates are not much lower. For example, with all six groups in the competition, 68.8% of the players in quintile 1 earn TOUR cards. Although not shown in the table, if players in quintile 1 are the only "good" players in

Q-School competition, their success rate increases to 71.1%.

Note that among players comparable to Tiger Woods and the players in the various quintile groups, very few are eliminated from Q-School competition in Pre-qualifying. However, starting in Stage 1, many players other than the Tiger-type player start to get knocked out. In fact, by the end of Stage 1, about 15% of the players in quintiles 4 and 5 are eliminated from Q-School. Generally, these players are being eliminated by lesser-skilled players who have experienced favorable random variation in their performance.

The tendency for favorable variation to be a determining factor in Q-School qualifying success could be reduced if the number of rounds of play per stage of Q-School were increased. Panel B of Table 10 summarizes success rates for a player comparable to Tiger Woods and players in quintiles 1-5, assuming that the number of rounds per stage is doubled. Here we observe higher qualifying success rates (e.g., 83.0% vs. 68.8% for quintile 1 and 42.3% vs. 28.7% for quintile 5), but rates that are lower than what we believe most would consider to be acceptable.

9.3. Success Rates for the Top 25 Players in the 2004-2008 PGA TOUR Sample

We now conduct a number of separate simulations where, in each simulation, we replace a single randomly selected player in pre-qualifying with a player whose scoring profile is identical to that of one of the 25 most highly-skilled players in the PGA TOUR sample. For the purposes of identifying the 25 players, a player's skill is defined as the average value of his spline-based mean 18-hole score as estimated over the 2004-2008 period. We also show simulation results for Q-School participants with scoring profiles equivalent to eight well-known players who are not as highly ranked.

Table 11 summarizes estimated success rates for each of the 33 players in each stage of Q-School, listed in the order of mean spline-based estimated skill. The standard deviation of η errors for each player is shown along with the first-order autocorrelation coefficient associated with his θ errors, denoted as ρ.

Among those entering Q-School in Pre-qualifying with skill comparable to a top-25 player, the median qualifying rate is only 64.1%. Individual qualifying success rates are not strictly monotonic with average spline values. For example, a player with Darren Clarke's scoring characteristics, the 16^{th} ranked player, has the lowest qualifying success rate, 48.5%, among the top 25. As such, the rates of qualifying success among the 33 players reflect a combination of mean skill, round-to-round variation in scoring, autocorrelation in θ errors, and variation in mean skill within each player's spline-based skill function. Generally, it appears that higher standard deviations of η errors and higher first-order autocorrelation in residual θ errors are associated with lower qualifying success rates.

We believe that most who follow golf would find it disturbing that a player with Phil Mickelson's scoring characteristics would have only an 87.8% chance of earning his TOUR card and that the next Ernie Els' would have only an 80.5% of qualifying for the TOUR. If a player like Mike Weir, the last listed among the top 25, enters Q-School in Pre-Qualifying, his qualifying success rate is estimated to be only 53.5%.

Players entering Pre-qualifying with scoring characteristics comparable to lesser-ranked players Camilo Vilegas, Boo Weekley, Chad Campbell, Lucas Glover, Sean O'Hair, and Brian Gay would have no more than a 43% chance of qualifying for the TOUR. But we believe these estimated success rates are quite realistic. For example, it took Sean O'Hair, ranked 15^{th} in the Official World Golf Rankings as of July 26, 2009 and 7^{th} in the Rankings among American golfers, six tries in Q-School between 1999 and 2004 before qualifying for the PGA TOUR. Since earning his TOUR card, O'Hair has won three PGA TOUR events. Similarly, Zach Johnson, ranked 23^{rd} and 10^{th} among American golfers, spent five years in Q-School between 1998 and 2002, with his best finish coming in 2002, a tie for 94^{th} in Stage 3. This earned Johnson a partial exemption on the 2003 Nationwide Tour, where he finished in first place, thereby earning the right to play on the PGA TOUR starting in 2004. Since then, he has won seven times on the PGA TOUR, including the 2007 Masters Tournament. Among highly-ranked young American golfers, including Lucas Glover, who spent two years in Q-School and one on the Nationwide Tour before joining the PGA TOUR in 2004 and winning the 2009 U.S. Open, Anthony Kim is the only player to have earned his TOUR card on his first attempt in Q-School.

10. Alternative Q-School Structures

Except for stage 3, each stage of Q-School is conducted in separate competitions, with winners at each competition site advancing to the next stage. For example, in our prototypical Q-School structure, summarized in Table 7, Pre-qualifying is conducted at six separate tournament sites, with a total of 397 players competing for 213 spots in Stage 1 competition. (In the literature of contest design, such a structure is known as a "subcontest"(Fu and Lu (2009), p. 147)). However, since the 213 winning players are determined in six separate Pre-qualifying competitions, the 213 players who played the best in Pre-qualifying do not necessarily qualify for Stage 1. In our first test of alternative Q-School structure, we estimate the probabilities that highly-skilled players are eliminated in Q-School competition because they happened to have been assigned to tournament sites for which they had lower chances of advancing.

To do so, we restructure our simulations into a series of "grand contests," so that each stage of Q-School is conducted at a single tournament site. Such a structure guarantees that the lowest-scoring players in each stage earn next-stage qualifying positions, although it does not guarantee that the most highly-skilled players in each stage get to advance. We run the simulations two ways, first without seeding Pre-qualifying with any special players with scoring characteristics comparable to specific PGA TOUR players and second, by including 'Tiger Woods' and players from quintiles 1-5, as in Table 10. As it turns out, the qualifying success rates are essentially the same as those obtained when Q-School is conducted using the prototype structure. Therefore, we conclude that using a number of separate tournament sites for stage qualifying is not a significant factor in strong players being eliminated in Q-School competition.

We also consider a Q-School structure in which all players compete in a single stage in multiple rounds on the same course. Such a structure would solve the problem of strong players being eliminated from competition prior to Stage 3 and also the minor problem, addressed above, associated with conducting Pre-qualifying through Stage 2 at multiple tournament sites.

Panel A of Table 12 shows qualifying success rates for skill groups 1-6 when Q-School is conducted with the prototypical structure summarized in Table 7. Throughout the table, success rates are shown with and without the inclusion of 'the next Tiger Woods' and players from quintiles 1-5. Panel B shows success rates when Q-School is conducted in a single stage on a single course over 18 rounds, the same number of total rounds in the prototype structure. In the single-stage structure, the equivalent to reaching Stage 3 is finishing among the lowest scoring 163 players, the same number of Stage 3 players in the prototype structure.

Comparing Panels A and B, we see that conducting Q-School over 18 rounds of play at a single tournament site, rather than over 18 rounds in four stages of single elimination competition, increases the probability that the 25 most highly-skilled players in Q-School will end up among the 25 players with the lowest scores. For example, when 'the next Tiger Woods' and players from quintiles 1-5 are not included, the probability of finishing among the lowest scoring 25 players increases from 33.0% to 43.2% for players in skill group 1. When 'Tiger' and his five counterparts are included, the probability increases from 38.0% to 49.3%.

We also consider the effects of doubling the number of tournament rounds from 18 to 36 (Panel C) and, again, from 36 to 72 (Panel D). With 72 rounds of play, 65.3% of those in skill group 1 finish among the lowest scoring 25 when 'Tiger' and counterparts are not competing, and 70.5% finish among the lowest scoring 25 when the Tiger group is competing. With or without the Tiger group, almost all players in group 1 finish among the lowest scoring 163, meaning that they would at least earn partially-exempt status on the Nationwide Tour.

Although qualifying success rates are improved when the number of rounds of tournament play rises, less than 2/3 of the players in skill group 1, the 25 most highly-skilled players in Q-School competition, would finish among the lowest scoring 25 after 72 rounds of play when 'Tiger' and counterparts are not included in the competition. Thus, even after 72 rounds of competition, it would not be possible to identify the 25 most highly skilled players with a high level of precision.

There is no practical way to conduct Q-School for 72 rounds on a single course. However, competition for a full year on the Nationwide Tour roughly approximates such a scenario, albeit with fewer competitors. In 2008, the 75 leading money winners on the Nationwide Tour averaged 74 18-hole rounds of play over the 2008 season, approximately the same as the 72 rounds in our alternative Q-School structure. PGA TOUR cards are awarded to the 25 leading money winners on the Nationwide Tour, which is not exactly the same as the 25 players with the lowest total scores over 72 rounds. Moreover, not all Nationwide players participate in the same events, they often miss cuts and are eliminated from individual tournaments, and money winnings are highly non-linear in scoring. Nevertheless, we believe that Nationwide competition is much closer in structure to our 72-round single-stage structure than regular Q-School structure conducted in four stages at multiple tournament sites. In this respect, it is not surprising that players who have earned their TOUR cards through Nationwide competition have tended to be more successful than those who earned their cards through Q-School.

11. Summary and Conclusions

This study uses simulation to estimate Type I and Type II error rates associated with the Q-School qualifying mechanism employed by the PGA TOUR to identify golfers of high skill to join the TOUR. We calibrate the simulations to reproduce success rates of golfers and standard deviations of scoring actually observed in the various stages of the Q-School qualifying process.

From simulation, we estimate that the current Q-School qualifying structure lead to high Type I and II errors. For example, we find that the median estimated qualifying success rate for golfers of skill comparable to that of the 25 most highly skilled players on the PGA TOUR is only 64.1%, but that many players of substantially less skill could qualify for the TOUR in a given year. Although skill plays a substantial role in determining qualifying success rates, for example, a player of Tiger Woods' ability, as estimated over the 2003-2009 period, would get through virtually all the time, random variation in scoring also plays a substantial role. This suggests that in any given year, one would expect a portion of those who play in the final stage of Q-School and end up qualifying for the TOUR would be less skilled relative to final stage participants who only qualify to play on the Nationwide Tour. This helps to explain why the long-term success of golfers who qualify for the PGA TOUR through Q-School is not as high as that of golfers who qualify for the Tour through the Nationwide Tour.

We explore the impact of alternative Q-School structures on the efficiency of the Q-School qualifying process. We find that a single-stage tournament conducted with substantially more rounds than the 18 in the current structure would be a more efficient mechanism for identifying high-skill golfers. Although impractical, this finding is consistent with the prediction in Ryvkin and Ortmann (2008) that a longer tournament should be the most efficient. This alternative structure resembles reasonably well the current Nationwide Tour, where the top 25 money winners over a season of nearly 30 tournaments automatically qualify to play the next season on the PGA TOUR.

The existing PGA TOUR qualification process has some features that we have not explored in detail here. As we noted in the introduction, the PGA TOUR seems to have recognized that Q-School and the Nationwide Tour both play an important role in identifying high-skill golfers for participation on the TOUR. Over the past few years, the PGA TOUR has changed the allocation of TOUR cards among Q-School and Nationwide qualifiers. Although the optimal allocation of

TOUR cards to each qualifying mechanism isn't known, an extension of the simulation framework developed in this study could prove useful for making this assessment.

Finally, it seems to us that through Q-School, and partially through the Nationwide Tour, the PGA TOUR is attempting to solve the general problem of human performance evaluation, given limited track records, that faces virtually all organizations. These evaluation activities are typically constrained by time, money, and other resources. But unlike Q-School and the Nationwide Tour, the systems of performance evaluation found in organizations may be far more complex and ambiguous. Compared with human performance evaluation activities found in academic departments, professional service firms, internship programs and other settings, the fixed set of rules associated with the Q-School selection metric provides a level of simplicity not likely to be found in the aforementioned settings. To the extent that our findings generalize into such settings, our results raise serious questions as to whether typical tournament-style selection mechanisms generate the right outcomes in noisy and ambiguous environments. We believe that further study of these issues is both intellectually challenging and of immense practical value.

Appendix

A. Scoring Distribution Modification Process for Non-PGA Tour Players

We summarize the process by which we produce scoring distributions for non-PGA Tour players in Q-School in (1) through (6) below.

$$h_{i,j} = \mu_j + \kappa_j \left(h_{i,j,k(i,j)} - \mu_j\right) + \phi_j \tag{1}$$

$$\mu_j = \begin{cases} \mu_2^*, \; for \; j = 0, 1, 2 \\ \mu_3^*, \; for \; j = 3 \end{cases} \tag{2}$$

$$\hat{\sigma}_{i,j,k(i,j)} = -1.5169 + 0.0599 h_{i,j,k(i,j)} \tag{3}$$

$$\hat{\sigma}_{i,j} = \alpha_j + \beta_j h_{i,j} \tag{4}$$

$$\xi_{i,j} = \hat{\sigma}_{i,j} / \hat{\sigma}_{i,j,k(i,j)} \tag{5}$$

$$\boldsymbol{\theta}_{i,j} = \boldsymbol{\theta}_{i,j,k(i,j)} \xi_{i,j} \tag{6}$$

In (1) through (6), the subscript i denotes a player in a simulation trial who enters Q-School in Stage $j = 0, ..., 3$, with Stage 0 denoting Pre-qualifying. If the player's entry stage is 0, 1, or 2, his level of skill and variation in scoring is based on that of a randomly drawn player (with replacement) among 161 in the sample of Stage 2 players coming from the Tour, denoted as player $k(i,j)$. The population mean of possible spline values that could be drawn from this group is μ_2^*. If the player enters Q-School in Stage 3, his level of skill and variation in scoring is based on that of a randomly drawn player among 86 in the sample of Stage 3 players coming from the Tour, also denoted as player $k(i,j)$. The population mean of possible spline values that could be drawn from this group is μ_3^*. For expositional convenience, henceforth we will refer to the appropriate Stage 2 or Stage 3 PGA Tour sample as player i's "base sample."

In (1), $h_{i,j,k(i,j)}$ denotes a mean skill level drawn randomly from the spline of the $k(i,j)^{th}$ player in the i^{th} competitor's base sample. This becomes the base skill level from which Q-School player i's mean skill level and variation in scoring is derived. When $\kappa_j > 1$, the $\mu_j + \kappa_j \left(h_{i,j,k(i,j)} - \mu_j\right)$ portion of (1) has the effect of widening the distribution of potential mean skill levels drawn initially from the base sample while preserving the population mean of the base sample from which the random skill level is drawn. Similarly, when $\kappa_j < 1$, the effect is to narrow the distribution of potential mean skill levels. The idea in using this spreading device is that for some entry stages, the distribution of mean skill levels might be more disperse than that associated with PGA Tour players entering Q-School in the stage from which the skill level of the i^{th} Q-School competitor is derived. We also include a mean displacement parameter, ϕ_j, which allows the overall population mean score for Q-School participants in a given entry stage to be higher or lower than that of its associated base sample.

The coefficients -1.5169 and 0.0599 in (3) are the intercepts and slopes of a least squares regression of the standard deviation of η errors on mean spline-based skill levels across the 303 players in the 2004-2008 PGA Tour sample, with adjusted $R^2 = 0.055$ and $p < 0.0001$. Consistent with this relationship, when a Q-School participant's simulated mean skill level is higher than the mean skill level drawn randomly from the spline of the $k(i,j)^{th}$ player in the i^{th} competitor's base sample (i.e., when $h_{i,j} > h_{i,j,k(i,j)}$), the θ and η errors which drive the random component associated with player i's simulated score should be more disperse than those of player $k(i,j)$. In (3) through (6) we adjust both η and θ errors in our simulations (since η errors are the primitives of θ errors) to capture this general relationship, while allowing the slope and intercept describing the relationship between the standard deviation of η errors and mean skill to vary by entry stage.

To simulate random variation in scoring for Q-School participant i, we select a starting θ error at random from the distribution of player $k(i,j)$'s θ errors estimated over the entire 2004-2008 PGA Tour sample period. We then select 28 of player $k(i,j)$'s η errors at random (with replacement). Using the initial θ error, the 28 randomly-selected modified η errors, and player $k(i,j)$'s first-order autocorrelation coefficient as estimated in connection with the Connolly-Rendleman (2008) model, we compute a sequence of 28 estimated θ errors. We then multiply the 28 θ errors by $\xi_{i,j}$, as in (6), to obtain a set of modified θ errors. We apply the last 18 modified θ errors to player i's randomly selected mean skill level, yielding a sequence of 18 simulated 18-hole scores for player i.

We model Pre-qualifying entrants as a bimodal group. The proportion γ of Pre-qualifying entrants is assumed to come from a group of lesser-skilled players who have only a minimal chance of qualifying for the Tour, while the proportion $1 - \gamma$ of Pre-qualifying entrants are assumed to be 'legitimate' players. In our simulations, both groups can have their own unique values of simulation parameters ϕ_0, α_0 and β_0. We assume that κ_0 is the same for both groups.

We calibrate our simulations by choosing a value of the parameter γ, together with parameter values κ_j, ϕ_j, α_j and β_j for entry stages 0,...,3, including separate parameter values of ϕ_0, α_0 and β_0 for the two groups of Pre-qualifying entrants, to produce simulated qualifying success rates and simulated standard deviations of 'neutral' scores per playing stage/entry stage category that closely approximate those observed empirically. Simulated scores are neutral with respect to both random round-course and player-course effects, which are very small, while the empirical distribution of Q-School scoring is neutral only with respect to round-course effects. Appendix Table 1 provides a summary of parameter values employed in the simulations.

Appendix Table 1: Parameters of Calibrated Simulation Model

	Entry Stage				
	Pre-Qualifying				
Parameter	Legitimate	Lesser-Skilled	Stage 1	Stage 2	Stage 3
ϕ	1.92	7.30	0.99	0.18	1.20
κ	1.16	1.16	1.225	1.50	1.10
α	-63.20	-67.71	-31.85	15.84	-1.52
β	0.8982	0.8982	0.4791	-0.1796	0.0599
γ	n/a	0.25	n/a	n/a	n/a

B. Sensitivity to Parameter Selection

In this section we explore the sensitivity of qualifying success rates to the selection of simulation parameters. Appendix Table 2 summarizes qualifying success rates for skill groups 1-6 for eight different parametrization schemes. In the simulations underlying Appendix Table 2, we employ the prototype Q-School structure summarized in Table 7 of the main paper and do not seed Pre-qualifying with any highly-skilled players from the PGA Tour such as Tiger Woods and players from quintiles 1-5.

Panel A of Appendix Table 2 shows the same success rates for skill groups 1-6 as reported in Table 10 of the main paper. In Panel B, all κ values are set to 1.0, which has the effect of not spreading the distributions of mean scores for non-PGA Tour players relative to those of the base PGA Tour samples from which they are drawn. β values are all set to 0.0597 and α values set to -1.5056, which has the effect of basing the relationship between the standard deviation of η errors and mean player skill on the least-squares regression, Equation 4, without modification. Although not shown in the table, this alternative parametrization substantially reduces the standard deviation of round-course-adjusted scores. Nevertheless, the qualifying success rates are hardly any different from those associated with the calibrated parameters.

In Panel C, the qualifying success rates reflect the calibrated parametrization, except that Pre-qualifying does not include a group of players who have no realistic chance of qualifying for the Tour. Again, the qualifying success rates are essentially the same as those associated with the calibrated parameters.

In Panel D, the qualifying success rates reflect the calibrated parametrization, except that all α values are increased by 1.0. This has the effect of increasing the standard deviation of round-course-adjusted scores. Here, the qualifying success rates are generally lower than those associated with the standard parametrization, since more lesser-skilled players can 'luck out' and qualify for the Tour.

In Panel E, the qualifying success rates reflect the calibrated parametrization, except that all mean displacement values, ϕ, are doubled. This creates a greater difference in skill among player groups, thereby increasing the success rates of the most highly-skilled players. Nevertheless, these success rates are not so much higher that we would change the basic conclusions of this study.

In Panel F, the qualifying success rates reflect the calibrated parametrization, except that all mean displacement values, ϕ, are halved. This creates a smaller difference in skill among player groups, thereby decreasing the qualifying success rates of the most highly-skilled players.

In Panel G, the qualifying success rates reflect the calibrated parametrization, except that all κ values are doubled. This has the effect of creating a significant amount of spread in the distributions of mean scores for non-PGA Tour players relative to those of the base PGA Tour samples from which they are drawn, thereby creating substantially greater differences in skill between highly-skilled and lesser-skilled players. As a result, qualifying success rates are substantially higher than those associated with the standard parametrization. But again, we do not believe that these success rates so high that we would change the basic conclusions of this study.

Finally, in Panel H, the qualifying success rates reflect the standard parametrization, except that all κ values are halved. This has the effect of creating substantially less spread between the distributions of mean scores for non-PGA Tour players relative to those of the base PGA Tour samples from which they are drawn. Here, the qualifying success rates are not qualitatively different from those of the base case.

Taken as a whole, our general conclusion that many highly-skilled players would not qualify for the PGA Tour through Q-School appears to be relatively insensitive to the calibration parameters employed in our simulations.

Appendix Table 2: Qualifying Success Rates with Changes in Simulation Parameters

Panel A: Original Calibration Parameters

Skill Group	Lowest Scoring 25	Lowest Scoring 75	Stage-3 Participant
1	33.0	59.9	72.7
2	17.4	42.7	60.4
3	12.9	35.5	55.6
4	8.1	26.0	45.4
5	5.4	19.2	36.9
6	3.4	13.7	29.4

Panel B: All $\kappa = 1$, $\beta = 0.0597$, $\alpha = -1.5056$

Skill Group	Lowest Scoring 25	Lowest Scoring 75	Stage-3 Participant
1.0	30.9	58.8	73.7
2.0	16.5	40.7	58.7
3.0	11.7	31.4	50.5
4.0	8.3	25.6	44.8
5.0	5.3	18.0	35.2
6.0	4.0	14.8	30.5

Panel C: Leave Out Bimodal Group ($\gamma = 0$)

Skill Group	Lowest Scoring 25	Lowest Scoring 75	Stage-3 Participant
1	32.9	60.0	72.8
2	17.4	42.6	60.3
3	13.0	35.4	55.4
4	8.1	26.0	45.5
5	5.5	19.4	36.9
6	3.4	13.7	29.3

Panel D: All α Increased by 1.0

Skill Group	Lowest Scoring 25	Lowest Scoring 75	Stage-3 Participant
1.0	26.0	51.4	65.1
2.0	14.3	37.4	54.7
3.0	10.7	31.6	50.5
4.0	7.4	23.4	41.3
5.0	5.3	17.8	33.7
6.0	3.6	12.7	26.4

Panel E: All ϕ Doubled

Skill Group	Lowest Scoring 25	Lowest Scoring 75	Stage-3 Participant
1	39.0	72.0	83.4
2	23.4	57.2	74.4
3	15.4	45.5	65.8
4	9.4	33.9	55.4
5	5.2	24.4	46.2
6	2.8	16.7	37.3

Panel F: All ϕ Halved

Skill Group	Lowest Scoring 25	Lowest Scoring 75	Stage-3 Participant
1.0	28.9	52.5	64.3
2.0	13.6	34.7	51.1
3.0	9.2	26.2	43.0
4.0	7.1	22.0	39.3
5.0	5.6	18.2	34.0
6.0	4.4	15.2	29.7

Panel G: All κ Doubled

Skill Group	Lowest Scoring 25	Lowest Scoring 75	Stage-3 Participant
1	54.7	80.5	86.3
2	16.3	52.6	67.1
3	8.6	35.9	55.4
4	5.2	25.1	45.9
5	3.5	19.8	41.1
6	2.5	15.7	36.7

Panel H: All κ Halved

Skill Group	Lowest Scoring 25	Lowest Scoring 75	Stage-3 Participant
1.0	31.9	61.3	76.9
2.0	16.5	40.1	58.6
3.0	11.2	30.7	50.2
4.0	7.6	23.5	41.8
5.0	5.3	18.6	36.1
6.0	4.0	15.4	32.6

Results based on 1,000 simulation trials using the prototype simulation structure summarized in Table 7 of the main paper without seeding Pre-qualifying with any highly-skilled players from the PGA Tour. Panel A results based on calibrated simulation parameters summarized in Appendix Table 1. Results in all other panels based on calibrated parameters except for changes indicated. Skill group 1 consists of the first 25 players per simulation trial based on the ranking of individual player skill. Group 2 consists of the next 25 players, etc. Only the first six groups are shown.

References

Appleton, David R., 1995, May the Best Man Win? *The Statistician* 44, no. 4:529 - 538.

Berry, Scott M. 2001. How Ferocious is Tiger. *Chance* 14, no. 3:51-56.

Berry, Scott M., C. Shane Reese and Patrick D. Larkey. 1999. Bridging Different Eras in Sports. *Journal of the American Statistical Association* 94, no. 447:661-676.

Birkinshaw, Julian. 2001. Strategies for Managing Internal Competition. *California Management Review* 44, no. 1:21 - 38.

Brown, Jennifer. 2010. Quitters Never Win: The (Adverse) Incentive Effects of Competing with Superstars. Unpublished manuscript, Kellogg Graduate School of Management, Northwestern University, Evanston, IL.

Caulkins, Jonathan P., Arnold Barnett, Patrick D. Larkey, Yuehong Yuan, Jesse Goranson. 1993. The On-Time Machines: Some Analyses of Airline Punctuality. *Operations Research* 41, no. 4:710-720.

Clarke, S. R., J. M. Norman, and C. B. Stride. 2009. Criteria for a Tournament: the World Professional Snooker Championship. *Journal of the Operational Research Society* 60, no. 12:1670 - 1673.

Connolly, Robert A. and Richard J. Rendleman, Jr.. 2008. Skill, Luck and Streaky Play on the PGA Tour. *The Journal of The American Statistical Association* 103, no. 1:74-88.

Connolly, Robert A. and Richard J. Rendleman. 2009. Dominance, Intimidation and 'Choking' on the PGA Tour. *Journal of Quantitative Analysis in Sports* 5, no. 3:1-32.

Davenport, Thomas H. and John C. Beck. 2000. Getting the Attention You Need. *Harvard Business Review* 78, no. 5:118 - 126.

David, H. A. 1998. *The Method of Paired Comparisons*. Oxford Press, New York.

Frick, Bernd. 2003. Contest Theory and Sport. *Oxford Review of Economic Policy* 19, no. 4:512 - 529.

Gibbons, Jean Dickinson, Ingram Olkin, and Milton Sobel. 1977. *Selecting and Ordering Populations: A New Statistical Methodology.* Wiley, New York.

Fu, Qiang and Jingfeng Lu. 2009. The Beauty of Bigness: On Optimal Design of Multi-Winner Contests. *Games and Economic Behavior* 66, no. 1:146-161.

Hvide Hans K. and Eirik G. Kristiansen. 2003. Risk Taking in Selection Contests. *Games and Economic Behavior* 42, no. 1:172 - 179.

Lazear, Edward P. and Sherwin Rosen. 1981. Rank-Order Tournaments as Optimum Labor Contracts. *Journal of Political Economy* 89, no. 5:841-864.

Ljung, G. M. and Box, G. E. P. 1978. On a Measure of Lack of Fit in Time Series Models. *Biometrika* 65, no. 2:297-303.

McGarry, T. and R. W. Schutz. 1997. Efficacy of Traditional Sport Tournament Structures. *Journal of the Operational Research Society* 48, no. 1:65 - 74.

McLaughlin, Kenneth J. 1988. Aspects of Tournament Models: A Survey. *Research in Labor Economics* 9, no. 1:225 - 256.

Narayana, T. V., 1979, *Lattice Path Combinatorics with Statistical Applications*. University of Toronto Press, Toronto.

Prendergrast, Canice. 1999. The Provision of Incentives in Firms. *Journal of Economic Literature* 37, no. 1:7 - 63.

PGA TOUR. 2008. Application to Enter 2008 PGA TOUR Qualifying Tournament (for the PGA TOUR and Nationwide Tour) Conducted by the PGA TOUR.

Rosen, Sherwin. 1986. Prizes and Incentives in Elimination Tournaments. *American Economic Review* 76, no. 4:701 - 715.

Ryvkin, Dmitry. 2010. The Selection Efficiency of Tournaments. *European Journal of Operational Research* 206, no. 3:667 - 675.

Ryvkin, Dmitry. and Andreas Ortmann. 2008. The Predictive Power of Three Prominent Tournament Formats. *Management Science* 54, no. 3:492-504.

Sirak, Ron. 2008. The Rich get Richer: The Money on Tour gets another Boost. No Wonder Tiger's Approaching $1 billion. *Golf Digest* 59, no. 2:96-100.

Wang, Y., 1998, Smoothing Spline Models with Correlated Random Errors. *The Journal of The American Statistical Association* 93, no. 1:341-348.

Table 1: World Golf Rankings of 1998-2008 PGA TOUR Qualifiers from Q-School and Nationwide Tour (7-26-2009)

World Golf Ranking	Q-School N	Q-School %	NW Tour N	NW Tour %
1-100	29	10.10	20	11.43
101-200	24	8.36	20	11.43
201-300	25	8.71	17	9.71
301-400	22	7.67	26	14.86
401-500	18	6.27	12	6.86
501-600	16	5.57	10	5.71
601-700	14	4.88	11	6.29
701-800	16	5.57	8	4.57
801-900	10	3.48	4	2.29
901-1000	9	3.14	7	4.00
Over 1000	104	36.24	40	22.86
Total	287	100.00	175	100.00

Table 2: Performance of Q-School Entrants, 2006-2008 Combined

Initial Stage	Final Stage Reached in 2006: Pre-Qual	Stage 1	Stage 2	Stage 3: NW (PE)	Stage 3: NW (FE)	Stage 3: PGA TOUR	Total
Pre-qual	552	505	121	9	4	0	1,191
Stage 1	0	1,288	615	114	57	25	2,099
Stage 2	0	0	288	72	49	45	454
Stage 3	0	0	0	48	43	24	115
Total	552	1,793	1,024	243	153	94	3,859

NW (PE) = Qualified for partially-exempt status on Nationwide Tour, NW (FE) = Qualified for fully-exempt status on Nationwide Tour, PGA TOUR = Qualified for fully-exempt status on PGA TOUR.

Table 3: Q-School Qualifying Success Rates by Playing Stage/Entry Stage

Entry Stage	Playing Stage 2006-2008 Combined			
	Pre-Qual	Stage 1	Stage 2	Stage 3
Pre-qual	0.537	0.210	0.097	0.000
Stage 1		0.386	0.242	0.128
Stage 2			0.366	0.271
Stage 3				0.209

Values in the table represent proportions of participants playing in a given stage who qualified for the next stage. The proportions shown advancing from Stage 3 are those who earned fully-exempt status on the PGA TOUR by finishing in the lowest scoring 40, 26 and 28 in 2006-2008, respectively.

Table 4: Normalized Mean Raw Scores by Playing Stage/Entry Stage

Entry Stage	Playing Stage 2006-2008 Combined			
	Pre-Qual	Stage 1	Stage 2	Stage 3
Pre-qual	3.457	2.444	2.215	0.749
Stage 1		1.496	1.265	0.482
Stage 2			0.790	0.054
Stage 3				0.000

In each panel, mean scores in each category are normalized relative to mean scores in Stage 3 for players who entered Q-School in Stage 3.

Table 5: Playing-Stage-Normalized Mean Round-Course-Adjusted Scores by Playing Stage/Entry Stage

Entry Stage	Playing Stage 2006-2008 Combined: Pre-Qual	Stage 1	Stage 2	Stage 3
Pre-qual	0.000	0.948	1.482	0.924
Stage 1		0.000	0.574	0.368
Stage 2			0.000	0.054
Stage 3				0.000

In each panel, mean scores in each category are normalized relative to mean scores in the category's playing stage for players who entered Q-School in the same stage.

Table 6: Standard Deviations of Round-Course-Adjusted Scores by Playing Stage/Entry Stage

Entry Stage	Playing Stage 2006-2008 Combined: Pre-Qual	Stage 1	Stage 2	Stage 3
Pre-qual	4.112	3.175	3.021	2.870
Stage 1		3.190	3.039	2.927
Stage 2			2.908	2.914
Stage 3				2.908

Table 7: 2006-2008 Prototype Q-School Structure Employed in Simulation Calibrations

Qualifiers	Pre-qual	Stage 1	Stage 2	Stage 3
Total next-stage qualifiers 2006-2008	639	945	375	94
Qualifiers/year*	213	315	125	31
Tournament sites	6	12	6	1
Qualifiers/tournament site	35.5	26.25	20.83	31

Participants	Pre-qual	Stage 1	Stage 2	Stage 3
Total participants 2006-2008	1,191	2,738	1,399	490
New entrants/year	397	700	151	38
Qualifiers/year from previous stage	0	213	315	125
Total participants/year*	397	913	466	163
Tournament sites	6	12	6	1
Participants/tournament site	66.17	76.08	77.67	163

*Values in first row of panel divided by 3, rounded to the nearest digit.

Table 8: Calibration: Simulation Results vs. Characteristics of Q-School Sample

Panel A: Qualifying Success Rates per Playing Stage/Entry Stage

Entry Stage	Simulation: Pre-Qual	Simulation: Stage 1	Simulation: Stage 2	Simulation: Stage 3	2006-2008 Q-School Sample: Pre-Qual	2006-2008 Q-School Sample: Stage 1	2006-2008 Q-School Sample: Stage 2	2006-2008 Q-School Sample: Stage 3	Simulation Minus Q-School Sample: Pre-Qual	Simulation Minus Q-School Sample: Stage 1	Simulation Minus Q-School Sample: Stage 2	Simulation Minus Q-School Sample: Stage 3
Pre-qual	0.537	0.209	0.096	0.037	0.537	0.210	0.097	0.000	0.000	-0.001	-0.001	0.037
Stage 1		0.386	0.241	0.125		0.386	0.242	0.128		0.000	-0.001	-0.003
Stage 2			0.368	0.269			0.366	0.271			0.002	-0.002
Stage 3				0.204				0.209				-0.005

Panel B: Standard Deviation of Round-Course-Adjusted Scores per Playing Stage/Entry Stage

Entry Stage	Simulation: Pre-Qual	Simulation: Stage 1	Simulation: Stage 2	Simulation: Stage 3	2006-2008 Q-School Sample: Pre-Qual	2006-2008 Q-School Sample: Stage 1	2006-2008 Q-School Sample: Stage 2	2006-2008 Q-School Sample: Stage 3	Simulation Minus Q-School Sample: Pre-Qual	Simulation Minus Q-School Sample: Stage 1	Simulation Minus Q-School Sample: Stage 2	Simulation Minus Q-School Sample: Stage 3
Pre-qual	4.097	3.176	3.016	2.874	4.112	3.175	3.021	2.870	-0.015	0.000	-0.005	0.005
Stage 1		3.169	3.046	2.940		3.190	3.039	2.927		-0.022	0.007	0.013
Stage 2			2.900	2.915			2.908	2.914			-0.008	0.001
Stage 3				2.909				2.908				0.001

Panel C: Playing-Stage-Normalized Round-Course-Adjusted Mean Scores per Playing stage/Entry stage

Entry Stage	Simulation: Pre-Qual	Simulation: Stage 1	Simulation: Stage 2	Simulation: Stage 3	2006-2008 Q-School Sample: Pre-Qual	2006-2008 Q-School Sample: Stage 1	2006-2008 Q-School Sample: Stage 2	2006-2008 Q-School Sample: Stage 3	Simulation Minus Q-School Sample: Pre-Qual	Simulation Minus Q-School Sample: Stage 1	Simulation Minus Q-School Sample: Stage 2	Simulation Minus Q-School Sample: Stage 3
Pre-qual	0.000	0.813	1.430	1.140	0.000	0.948	1.482	0.924	0.000	-0.135	-0.052	0.216
Stage 1		0.000	0.612	0.358		0.000	0.574	0.368		0.000	0.038	-0.010
Stage 2			0.000	-0.341			0.000	0.054			0.000	-0.394
Stage 3				0.000				0.000				0.000

Results based on 1,000 simulation trials, using the simulation parameters summarized in Appendix Table 1.

Table 9: Qualifying Success Rates by Skill Group

	Percentages per Category by Skill Group			Number of Players per Category by Skill Group					
	Lowest	Lowest	Stage 3	Lowest-Scoring 25		Lowest-Scoring 75		Stage 3 Participant	
Skill Group	Scoring 25	Scoring 75	Participant	Players	Cumul.	Players	Cumul.	Players	Cumul.
1	33.0	59.9	72.7	8.2	8.2	15.0	15.0	18.2	18.2
2	17.4	42.7	60.4	4.3	12.6	10.7	25.7	15.1	33.3
3	12.9	35.5	55.6	3.2	15.8	8.9	34.5	13.9	47.2
4	8.1	26.0	45.4	2.0	17.8	6.5	41.0	11.4	58.5
5	5.4	19.2	36.9	1.4	19.2	4.8	45.8	9.2	67.8
6	3.4	13.7	29.4	0.9	20.0	3.4	49.3	7.3	75.1
7	2.7	11.8	26.7	0.7	20.7	3.0	52.2	6.7	81.8
8	1.9	9.8	22.9	0.5	21.2	2.5	54.7	5.7	87.5
9	1.7	8.6	22.1	0.4	21.6	2.1	56.8	5.5	93.0
10	1.7	8.0	20.8	0.4	22.0	2.0	58.8	5.2	98.2
11	1.4	6.7	18.1	0.4	22.4	1.7	60.5	4.5	102.8
12	1.2	6.3	18.0	0.3	22.7	1.6	62.1	4.5	107.3
13	1.2	5.6	16.7	0.3	23.0	1.4	63.5	4.2	111.4
14	1.0	4.8	15.5	0.2	23.2	1.2	64.7	3.9	115.3
15	0.9	4.6	15.0	0.2	23.5	1.2	65.8	3.7	119.1
16	0.9	4.4	14.8	0.2	23.7	1.1	66.9	3.7	122.8
17	0.8	4.0	14.2	0.2	23.9	1.0	67.9	3.6	126.3
18	0.6	4.0	14.3	0.2	24.0	1.0	68.9	3.6	129.9
19	0.6	3.3	12.7	0.1	24.2	0.8	69.7	3.2	133.1
20	0.5	2.7	11.9	0.1	24.3	0.7	70.4	3.0	136.0
21	0.1	0.6	3.4	0.7	25.0	4.6	75.0	27.0	163.0

Results based on 1,000 simulation trials, using the simulation parameters summarized in Appendix Table 1. Skill group 1 consists of the first 25 players per simulation trial based on the ranking of individual player skill. Group 2 consists of the next 25 players, etc., with group 20 consisting of players ranked 476-500. Group 21 consists of the last $1,286 - 500 = 786$ players.

Table 10: Estimated Probabilities of Success at Each Stage of Q-School when 'Tiger' and Quintile Players Enter Q-School in Pre-Qualifying

Panel A: Standard Q-School Structure

	Stage			
Player	Pre-Q	1	2	3
Tiger	1.000	1.000	0.995	0.993
Quintile 1	0.998	0.952	0.846	0.688
Quintile 2	0.998	0.920	0.740	0.497
Quintile 3	0.997	0.899	0.643	0.408
Quintile 4	0.996	0.875	0.622	0.357
Quintile 5	0.994	0.838	0.579	0.287

Panel B: Number of Rounds in Each Stage of Q-School is Doubled

	Stage			
Player	Pre-Q	1	2	3
Tiger	1.000	1.000	1.000	1.000
Quintile 1	1.000	0.988	0.924	0.830
Quintile 2	0.999	0.975	0.861	0.693
Quintile 3	1.000	0.962	0.817	0.566
Quintile 4	1.000	0.941	0.762	0.493
Quintile 5	0.999	0.943	0.743	0.423

Results based on 1,000 simulation trials, using the simulation parameters summarized in Appendix Table 1.

Table 11: Estimated Probabilities of Success at Each Stage of Q-School for Selected Players from the 2004-2008 PGA TOUR Sample

Skill Rank	Player	Average Spline	Standard Deviation η	ρ	Estimated Probabilities of Success Pre-Q	Stage 1	Stage 2	Stage 3
1	Tiger Woods	68.072	2.494	0.009	1.000	1.000	0.996	0.995
2	Phil Mickelson	69.186	2.672	0.004	1.000	0.985	0.935	0.878
3	Vijay Singh	69.212	2.586	0.052	1.000	0.984	0.922	0.858
4	Ernie Els	69.377	2.682	0.076	1.000	0.969	0.900	0.805
5	Jim Furyk	69.474	2.286	-0.009	1.000	0.993	0.931	0.864
6	Sergio Garcia	69.609	2.506	0.022	0.999	0.982	0.923	0.832
7	Luke Donald	69.669	2.634	0.021	1.000	0.982	0.894	0.799
8	Adam Scott	69.703	3.037	0.055	0.995	0.940	0.829	0.706
9	Retief Goosen	69.917	2.733	0.010	0.999	0.954	0.838	0.676
10	Stewart Cink	69.936	2.441	-0.004	0.999	0.974	0.880	0.742
11	Padraig Harrington	70.032	2.845	0.092	1.000	0.943	0.760	0.590
12	David Toms	70.075	2.575	0.140	0.998	0.945	0.795	0.636
13	K.J. Choi	70.137	2.605	0.016	0.999	0.949	0.810	0.641
14	Stephen Ames	70.145	2.669	-0.033	0.999	0.958	0.806	0.615
15	Scott Verplank	70.157	2.516	0.048	0.998	0.945	0.818	0.648
16	Darren Clarke	70.165	3.267	0.194	0.987	0.847	0.654	0.485
17	Stuart Appleby	70.185	2.825	-0.015	0.998	0.946	0.792	0.594
18	Anthony Kim	70.195	2.791	-0.036	1.000	0.935	0.786	0.616
19	Geoff Ogilvy	70.200	2.721	-0.017	0.998	0.937	0.798	0.619
20	Robert Allenby	70.209	2.561	-0.043	0.999	0.956	0.832	0.635
21	Jay Haas	70.227	2.581	-0.022	0.998	0.924	0.757	0.578
22	Justin Rose	70.228	2.797	-0.042	1.000	0.942	0.787	0.602
23	Loren Roberts	70.249	2.573	-0.207	0.999	0.968	0.868	0.676
24	Kenny Perry	70.286	2.731	0.010	0.998	0.935	0.748	0.555
25	Mike Weir	70.290	2.613	0.172	1.000	0.921	0.728	0.535
26	Zach Johnson	70.292	2.583	-0.009	1.000	0.949	0.775	0.580
36	Ian Poulter	70.525	2.561	-0.069	1.000	0.940	0.753	0.539
41	Camilo Villegas	70.613	2.756	0.070	0.996	0.873	0.649	0.424
45	Boo Weekley	70.620	2.808	0.036	1.000	0.893	0.668	0.427
51	Chad Campbell	70.656	2.775	0.033	0.999	0.892	0.670	0.430
59	Lucas Glover	70.757	2.730	0.086	0.993	0.884	0.657	0.397
62	Sean O'Hair	70.774	2.735	0.143	0.996	0.865	0.620	0.369
88	Brian Gay	70.969	2.580	0.016	0.998	0.855	0.551	0.291

Results based on 1,000 simulation trials, using the simulation parameters summarized in Appendix Table 1. ρ denotes the estimated first-order autocorrelation coefficient associated each player's residual θ errors.

Table 12: Qualifying Success Rates in Single-Stage Q-School Competition

Panel A: Standard Staging Structure

Skill Group	Without Tiger and Quintile Players: Lowest Scoring 25	Lowest Scoring 75	Stage 3 Participant	With Tiger and Quintile Players: Lowest Scoring 25	Lowest Scoring 75	Stage 3 Participant
1	33.0	59.9	72.7	38.0	63.7	74.8
2	17.4	42.7	60.4	16.8	43.3	60.5
3	12.9	35.5	55.6	12.1	35.5	56.1
4	8.1	26.0	45.4	7.9	26.7	46.7
5	5.4	19.2	36.9	5.0	19.7	38.4
6	3.4	13.7	29.4	3.3	13.8	30.2

Panel B: Single Stage, 18 Rounds

Skill Group	Without Tiger and Quintile Players: Lowest Scoring 25	Lowest Scoring 75	Lowest Scoring 163	With Tiger and Quintile Players: Lowest Scoring 25	Lowest Scoring 75	Lowest Scoring 163
1	43.2	72.8	90.2	49.3	77.7	92.4
2	19.1	49.3	75.4	18.3	50.4	77.0
3	11.5	35.8	63.9	10.6	36.1	65.2
4	6.8	26.0	53.2	6.3	26.0	54.1
5	4.0	18.4	44.3	3.4	18.1	44.4
6	2.8	13.7	36.0	2.2	13.4	36.4

Panel C: Single Stage, 36 Rounds

Skill Group	Without Tiger and Quintile Players: Lowest Scoring 25	Lowest Scoring 75	Lowest Scoring 163	With Tiger and Quintile Players: Lowest Scoring 25	Lowest Scoring 75	Lowest Scoring 163
1	55.5	86.8	97.4	61.4	90.0	98.1
2	21.9	63.5	89.5	20.0	65.0	90.9
3	10.7	44.4	78.4	9.4	45.1	79.8
4	5.0	29.2	65.5	4.2	29.2	66.4
5	2.5	19.7	53.7	2.0	19.1	54.4
6	1.4	13.0	43.1	1.1	12.5	43.5

Panel D: Single Stage, 72 Rounds

Skill Group	Without Tiger and Quintile Players: Lowest Scoring 25	Lowest Scoring 75	Lowest Scoring 163	With Tiger and Quintile Players: Lowest Scoring 25	Lowest Scoring 75	Lowest Scoring 163
1	65.3	95.4	99.8	70.5	96.9	99.9
2	22.5	77.2	97.4	20.2	78.8	98.0
3	8.3	53.3	90.6	6.3	54.2	91.8
4	2.5	31.5	78.7	2.0	31.6	80.2
5	0.8	16.9	64.1	0.6	16.4	65.5
6	0.3	8.8	48.7	0.2	8.1	49.4

Results based on 1,000 simulation trials, using the simulation parameters summarized in Appendix Table 1. In Sections B-D, all Q-School participants compete together at a single tournament site for the number of rounds shown. Skill group 1 consists of the first 25 players per simulation trial based on the ranking of individual player skill. Group 2 consists of the next 25 players, etc. Only the first six groups are shown.

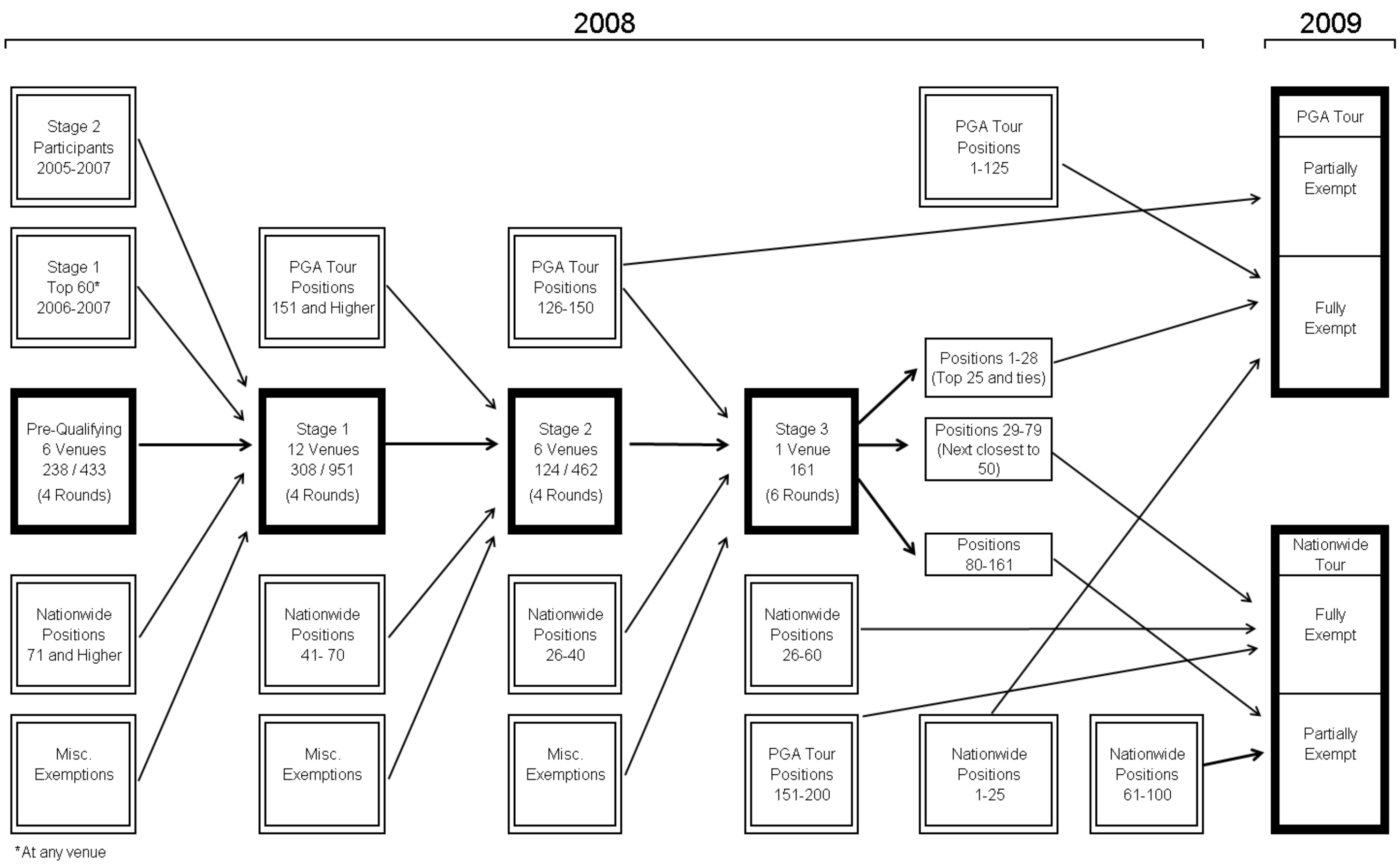

Figure 1: Relationship among Q-School, Nationwide Tour and PGA TOUR at the end of the 2008 season. Dark framed boxes indicate Q-School qualifying stages. Within these boxes N/M indicates that N players out of a total of M participants qualified for the next stage of Q-School. Q-School Pre-qualifying and stages 1 and 2 consist of four 18-hole rounds. Stage 3 consists of six 18-hole rounds. Positions for the PGA and Nationwide Tours are based on 2008 Official Money winnings. Positions for Q-School are based on total score.

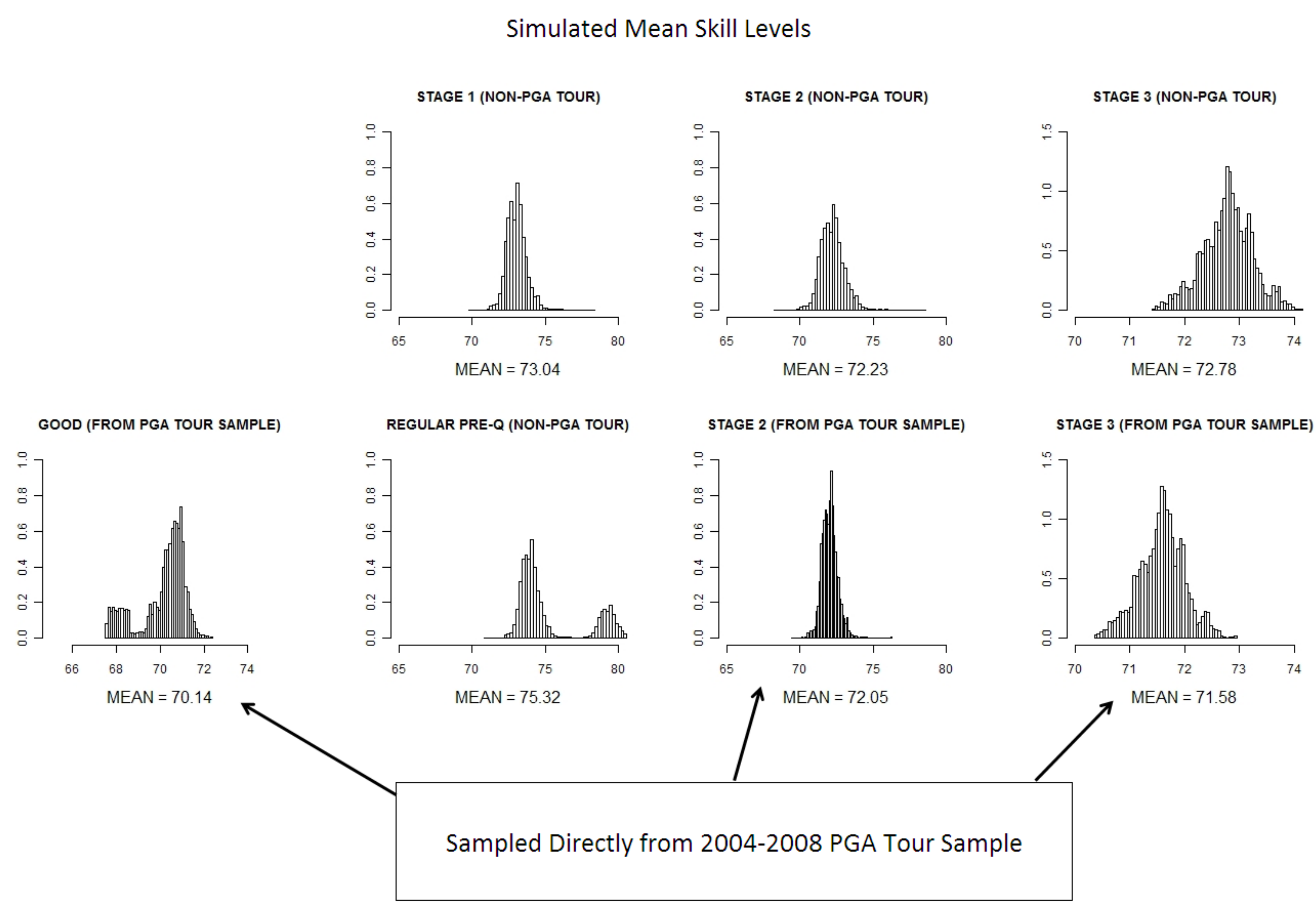

Figure 2: Distributions of simulated mean skill levels.

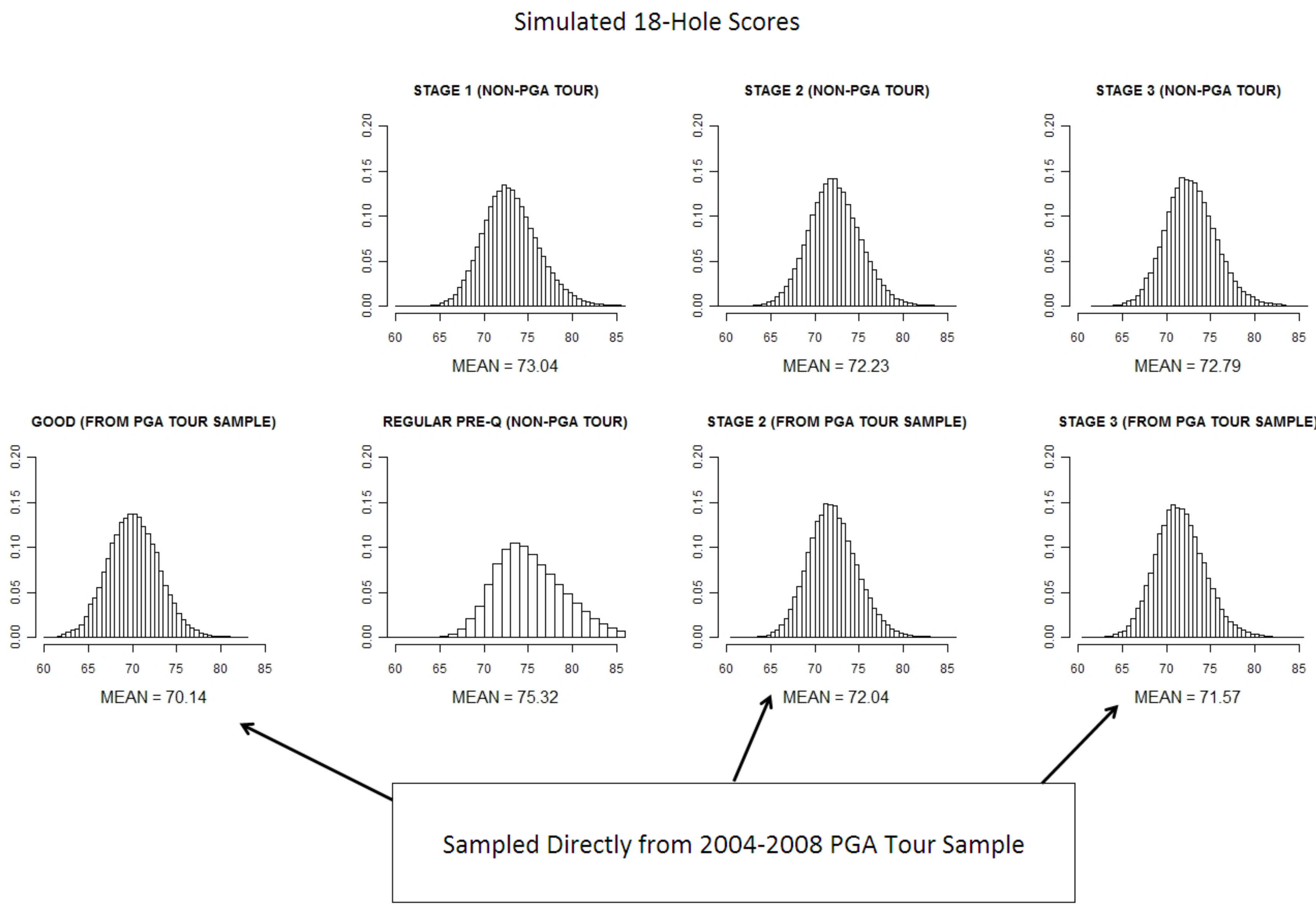

Figure 3: Distributions of simulated 18-hole scores.